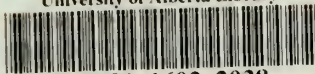


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Outcomes with Assessment Standards for Applied Mathematics 20

August 2002

This document is intended to assist teachers with the provincial implementation of Applied Mathematics 20.

This document replaces the February 2000 discussion draft and the August 2001 revised draft, and reflects the 2002 Program of Studies.



Learning and Teaching Resources Branch

Alberta Learning Cataloguing In Publication Data

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The primary intended audience for this document is:

<i>Administrators</i>	
<i>Counsellors</i>	
<i>General Audience</i>	
<i>Parent School Councils</i>	
<i>Parents</i>	
<i>Students</i>	
<i>Teachers</i>	✓

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INTRODUCTION

Applied Mathematics 20 became available for optional implementation in September 1999. Teachers field validating Applied Mathematics 20, and preparing for provincial implementation in September 2001, expressed a need for a common understanding of the curriculum and assessment standards for this new course. In response to this need, and in keeping with Alberta Learning's goal of establishing and effectively communicating clear outcomes and high standards for each area of learning, this standards document was developed.

PURPOSE

Outcomes with Assessment Standards for Applied Mathematics 20 contains samples of tasks linked to the specific outcomes from the program of studies, along with information and commentaries about standards. Its purpose is to provide teachers of Applied Mathematics 20 with clearly stated standards to use as guidelines in their classroom instruction and assessment practices. This document is not intended as an assessment tool or test package, but only as a guide that provides models of tasks that can be assigned or discussed in the classroom. The sample tasks in this document are intended for teacher use, but they also can be used to communicate to the broader educational community examples of acceptable student work and excellent student work in Applied Mathematics 20.

DEFINITIONS AND TERMINOLOGY

Standards

A *standard* is a reference point used in planning and evaluation. In evaluating educational performance, the following standards apply:

- *curriculum and assessment standards* apply to the assessment of individual students
- *achievement standards* apply to the assessment of student populations.

In this document, only curriculum and assessment standards are discussed.

Curriculum Standards

Curriculum standards are outcomes for a course or grade level of a program. The curriculum standards for Applied Mathematics 20 are defined by the general and specific outcomes outlined in the program of studies.

Outcomes

General outcomes are concise statements identifying what it is that students are expected to know and be able to do upon completion of a course or grade level of a program.

Specific outcomes are statements identifying the component knowledge, skills and attitudes of a general outcome. Specific outcomes identify a range of contexts in which the general outcomes apply.

Assessment Standards

Assessment standards are the criteria used for judging individual student achievement relative to the curriculum standards.

Assessment Instrument

An *assessment instrument* is a group of questions or tasks given to students to ascertain if they have met the requirements of the acceptable standard or the standard of excellence.

Blueprint

A *blueprint* is a classification of the questions or tasks making up a particular assessment instrument or an assessment plan. The classification may be in terms of standards (acceptable or excellent), content sections (topics or titles), mathematical understandings (concepts, procedures, problem-solving skills), or any other such classification.

Conceptual Understanding

Tasks assessing *conceptual understanding* are characterized by the use of such action verbs as defining, demonstrating, describing, developing, establishing, explaining, giving, identifying, illustrating, linking, recognizing and representing.

Procedural Knowledge

Tasks assessing *procedural knowledge* are characterized by the use of such action verbs as approximating, calculating, constructing, estimating, factoring, locating, rationalizing, simplifying, sketching, solving, using and verifying. Guided project work fits into this category.

Problem-solving Skills

Problem-solving skills refer to the construction of models describing applications in a broad range of contexts. They refer to the combination of algorithms to determine more complicated solutions and the use of appropriate technology to approximate difficult models or to carry out long calculation procedures. They also refer to the investigation of phenomena, the collection and interpretation of data in various contexts, and the transfer of knowledge from mathematics to other areas of human endeavour. Problem-solving tasks are characterized by the use of such action verbs as adapting, analyzing, combining, communicating, comparing, connecting, constructing, deriving, evaluating, fitting, investigating, justifying, modelling, proving, reconstructing and translating.

STANDARDS FOR APPLIED MATHEMATICS 20

The content specified in the program of studies for Applied Mathematics 20 contains the knowledge, skills and attitudes that all students are expected to acquire. As well, there are specific outcomes for problem solving and the use of technology. For complete details of the Applied Mathematics 20 program structure, refer to the program of studies.

Applied Mathematics 20 is designed to follow directly from Applied Mathematics 10, so students taking Applied Mathematics 20 are presumed to have reached the acceptable standard or better in the outcomes of Applied Mathematics 10. In particular, students will have facility in using the graphing calculator to graph functions and will be able to use a variety of measurement instruments. They have the ability to use coordinate methods to determine the properties of lines and line segments, they use statistically reliable techniques to obtain sample data, and they can solve a variety of trigonometry problems.

The assessment standards for Applied Mathematics 20 provide descriptions of student performances at an acceptable and an excellent level. Student performance should be measured on a range of tasks, some of which are routine and obvious tasks in familiar contexts, and others which are nonroutine tasks in unfamiliar contexts.

Acceptable Standard

The *acceptable standard* of achievement in Applied Mathematics 20 is met by students who receive a course mark between, and including, 50 per cent and 79 per cent. Typically, these students have gained new skills and a basic knowledge of the concepts and procedures relative to the general and specific outcomes defined in the Applied Mathematics 20 program of studies. These students can apply this knowledge to a limited range of familiar problem contexts.

Standard of Excellence

The *standard of excellence* for achievement in Applied Mathematics 20 is met by students who receive a course mark at, or above, 80 per cent. Typically, these students have gained a breadth and depth of knowledge regarding the concepts and procedures, as well as the ability to apply this knowledge to a broad range of familiar and unfamiliar problem contexts. This standard signifies high-quality performance relative to the general and specific outcomes in the Applied Mathematics 20 program of studies.

Description of Standards

The following statements describe what is expected of Applied Mathematics 20 students who meet the *acceptable standard* or the *standard of excellence* on independent work. The statements represent the standards against which student achievement is measured.

Acceptable Standard

Students who meet the *acceptable standard* in Applied Mathematics 20 consistently perform acceptable work on routine and obvious tasks in familiar contexts.

These students have a basic understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in one or two ways, and can do simple conversions from one mode to another. They perform the mathematical operations and procedures that are fundamental to applied mathematics at the 20 level, and apply what they know in daily living contexts.

Standard of Excellence

Students who meet the *standard of excellence* in Applied Mathematics 20 consistently perform excellent work on routine and obvious tasks in familiar contexts, and acceptable work on nonroutine tasks in unfamiliar contexts.

These students have a comprehensive understanding of the concepts and procedures outlined in the program of studies. They demonstrate their understanding in a variety of modes and can translate from one mode to another. They perform the mathematical operations and procedures that are fundamental to applied mathematics at the 20 level, apply what they know in daily living contexts and provide alternative solution procedures to verify results.

To meet the *acceptable standard*, students communicate about mathematical situations in an understandable way, using appropriate everyday and mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar contexts. These students adapt mathematical models from one situation to another.

Students meeting the *acceptable standard* are able to follow directions in such documents as calculator manuals, computer software documentation and measuring instrument manuals to perform familiar tasks. They know when to use such directions and can produce reliable results for familiar tasks.

Students meeting the *acceptable standard* have a positive attitude toward mathematics and a sense of personal competence in using mathematics. They demonstrate confidence when using common mathematical procedures and when applying problem-solving strategies in familiar settings.

To meet the *standard of excellence*, students communicate about mathematical situations in a clear way, using numbers, diagrams and appropriate mathematical terms. They understand mathematical questions containing objects, diagrams or numbers in familiar and unfamiliar contexts. These students construct mathematical models by translating words into suitable numbers, diagrams, tables, equations and variables.

Students meeting the *standard of excellence* are able to follow directions in such documents as calculator manuals, computer software documentation and measuring instrument manuals to perform familiar and novel tasks. They know when to use and when to modify such directions, and can produce reliable results for both familiar and novel tasks.

Students meeting the *standard of excellence* have a positive attitude toward mathematics and show confidence in using mathematics meaningfully. They are self-motivated risk takers who persevere when solving novel problems. They take initiative in trying new methods and are creative in their approach to problem solving.

ASSESSMENT PLANS, SUGGESTED WEIGHTINGS AND BLUEPRINTS

These plans and blueprints provide general guidelines for the overall assessment of Applied Mathematics 20.

Suggested Weightings by Topic			
Topic	Specific Outcomes (Summary)	Addison-Wesley <i>Applied Mathematics 11</i> Resource	Suggested Per Cent Weighting
Graphing and Design	1.1 Extract information from given graphs of discrete or continuous data. 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation.	Chapter 1 in Source Book Projects 1–6 in Project Book. [To cover all of the specific outcomes in this topic, Project 1 must be completed.]	15–17%
Regression and Nonlinear Equations	2.1 Solve nonlinear equations, using a graphing tool. 2.2 Determine the characteristics of the graph of a quadratic function. 2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions. 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems. 2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions.	Chapters 1 and 2 in Source Book Project 7 in Project Book	12–15%
Linear Systems and Programming	3.1 Graph linear inequalities in two variables. 3.2 Solve systems of linear equations in two variables. 3.3 Use expressions containing variables to describe problem contexts and solutions. 3.4 Solve, graphically, systems of linear inequalities, in two variables, using technology. 3.5 Design and solve linear and nonlinear systems, in two variables, to model problem situations. 3.6 Apply linear programming to find optimal solutions to decision-making problems.	Chapters 2 and 3 in Source Book Projects 8 and 9 in Project Book	18–20%
Finance	4.1 Solve consumer problems. 4.2 Reconcile financial statements. 4.3 Solve budget problems, using graphs and tables to communicate solutions. 4.4 Solve investment and credit problems involving simple and compound interest.	Chapter 4 in Source Book Projects 10–15 in Project Book	10–12%
Circle Geometry and Design	5.1 Use technology and measurement to confirm and apply circle properties. 5.2 Use properties of circles and polygons to solve design and layout problems.	Chapter 5 in Source Book Projects 16–19 in Project Book. [To cover all of the specific outcomes in this topic, one of projects 17, 18 and 19 must be completed.]	16–18%
Measurement and Design	6.1 Enlarge or reduce a dimensioned object, according to a specified scale. 6.2 Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors. 6.4 Design an appropriate measuring process or device to solve a problem.	Chapter 6 in Source Book Projects 20–25 in Project Book. [To cover all of the specific outcomes in this topic, one of projects 24 and 25 must be completed.]	16–18%

Suggested Per Cent Weightings by Standard and Skill				
	Conceptual Understanding	Procedural Knowledge	Problem-solving Skills	Totals
Acceptable Standard	$\cong 20\%$	$\cong 30\%$	$\cong 25\%$	75–80%
Standard of Excellence	$\cong 10\%$	$\cong 5\%$	$\cong 10\%$	20–25%
Totals	25–30%	35–40%	30–35%	100%

TECHNOLOGY REQUIREMENTS

To meet the outcomes of Applied Mathematics 20, students will need access to a graphing calculator and a computer with a spreadsheet program. In addition, measuring devices in both metric and imperial units are required. In most classrooms, students will not use technology on a daily basis; however, there are some topics of study that require greater access than others. The following table contains descriptions of the technology-related knowledge and skills expected of students in each of the topic areas, with suggested time requirements.

Topic	Estimated Time Requirement for Access to Technology	Technology-related Knowledge and Skills
Graphing and Design	3–4 hours	<i>Students will be expected to:</i> - display data in various formats on a graphing calculator - enter data in lists - choose the appropriate window settings - plot the appropriate graph - explore the chart menu on a computer spreadsheet program (optional) - search the Internet (optional)
Regression and Nonlinear Equations	8–10 hours	- graph a scatterplot on a graphing calculator - enter data in lists - choose the appropriate window settings - plot the graph - use the zoom feature - graph an equation, using the graphing calculator - enter the equation in the $y =$ form - plot the graph - display a table or use the trace function to estimate the domain and range - use the regression menu to determine appropriate equations - use a calculator-based ranger (CBR) or a calculator-based laboratory (CBL) to collect data (optional)
Linear Systems and Programming	10–12 hours	- graph a scatterplot on a graphing calculator - enter data in lists - choose the appropriate window settings - plot the graph - use the zoom feature - graph an equation, using the graphing calculator - enter the equation in the $y =$ form - plot the graph - display a table or use the trace function to estimate the domain and range - display graphs with shading for linear inequalities
Finance	5–6 hours	- use a computer spreadsheet program, including the application of user-defined functions - use a financial solver program on the graphing calculator - use commercial budget software programs (optional)
Circle Geometry and Design	3–4 hours	- use dynamic geometry software to explore the properties of circles and polygons - use a compass, protractor and straightedge to explore the properties of circles and polygons
Measurement and Design	1–2 hours	use metric and imperial measurement devices

MATHEMATICAL CONVENTIONS USED IN THE DESCRIPTION OF SAMPLE TASKS

Approximations, Estimates and Exact Values

An *approximation* to an exact value is a decimal representation, correct to several decimal places, used in a problem. Usually, these approximations result from calculator usage. For example, $\sqrt{3}$ may be replaced by the approximation 1.7321... during the computational phase of a problem.

An *estimate* of an exact value is an approximation that can be derived mentally, and used as part of a mental calculation. For example, the conjecture $\sqrt{2} + \sqrt{8} = \sqrt{10}$ can be shown to be false by using the estimates 1.5, 2.8 and 3.1 for $\sqrt{2}$, $\sqrt{8}$ and $\sqrt{10}$.

The *exact value* of a rational number is of the form $\frac{p}{q}$, where p and q are integers. Unless the decimal representation terminates, the decimal representation will be an approximation. So $-\frac{5}{3}$ is exact, while $-1.666...$ is an approximation.

The *exact value* of an irrational number is expressed in such forms as $\sin 35^\circ$, $\sqrt{3}$ or $\frac{\pi}{6}$. The decimal representation will always be an approximation. So $\sin 35^\circ$ is exact, while 0.573576... is an approximation.

Calculator Use in Multipart Problems

The general rule is to keep the calculator running from beginning to end, using all the available decimal places. All answers, both intermediate and final, are reported to the appropriate number of significant figures. However, rounded intermediate answers are **not** to be used as inputs into subsequent calculations. For example, a right-angled triangle with a measured hypotenuse of 11.9 cm and a shortest side of 6.1 cm would have the third side reported as 10.2 cm. However, if the length of the third side were used in the second part of a calculation, the more precise value 10.21763182... would be used as the input value.

Measured and Exact Input Data

If the *input data is measured*, all output data should be recorded in decimal form to the same quality as the least reliable part of the input data. For example, if the input data had 57.235 g for the mass and 17 mL for the volume, the density would be recorded as 3.4 g/mL with two significant digits. As a matter of convention, trigonometric problems with lengths to three significant digits yield angles to one-tenth of a degree, or one-hundredth of a radian.

If the *input data is exact*, all output data should be recorded in fractional form if at all possible. For example, the slope of the line joining the points (6, 11) and (17, 32) should be recorded as $\frac{21}{11}$, which is exact, rather than the decimal form 1.9090....

For Applied Mathematics 20, all trigonometric data can be assumed to be measured. In addition, most estimated regression coefficients used in calculations of correlation coefficients and predicted y -values can be assumed to be measured. All input data in coordinate geometry, algebraic expressions, relations and functions can be assumed to be exact. Most of the input data in number tables and number patterns can be assumed to be exact, except that money outputs are rounded to the nearest dollar or the nearest cent, depending on the context.

If there is any doubt whether any item of input data is exact or measured, assume it to be measured if it is part of an Applied Mathematics 20 problem.

STANDARDS LINKED TO OUTCOMES

The remainder of this document consists of several components that, taken together, provide guidance to teachers about the scope, depth and breadth of Applied Mathematics 20.

Topics

Topics are the mathematical themes that are used to organize concepts. The general and specific outcomes and the associated tasks in this part of the document are grouped according to topics.

General and Specific Outcomes

The general and specific outcomes for Applied Mathematics 20 identify what students are expected to know and be able to do upon completion of the course.

Strands

Strands are the underlying themes running throughout all of the mathematics programs. The Applied Mathematics 20 course includes:

- Number (Number Operations)
- Patterns and Relations (Variables and Equations)
- Patterns and Relations (Relations and Functions)
- Shape and Space (Measurement)
- Shape and Space (3-D Objects and 2-D Shapes)
- Statistics and Probability (Data Analysis)

Mathematical Processes

These are the critical components that students must encounter in a mathematics program in order to achieve the goals of mathematics education and to encourage lifelong learning in mathematics. One or more of the following mathematical processes is emphasized for each specific outcome.

[C] Communication	[R] Reasoning
[CN] Connections	[T] Technology
[E] Estimation and Mental Mathematics	[V] Visualization
[PS] Problem Solving	

More information on the general and specific outcomes, strands and mathematical processes can be found in the *Mathematics Applied and Pure Programs Program of Studies*.

Notes

Each specific outcome is followed by a note that provides additional information about the intended depth and breadth for that outcome. Teachers can refer to these for guidance when planning classroom activities and assessments that address the specific outcome at an appropriate level for the course.

Sample Tasks

Linked to each specific outcome are sample tasks. These consist of questions, problems or activities for classroom use with students. Teachers may use these directly or as models for developing their own tasks for group or independent work in the classroom, or as part of formative and summative assessment of students. The sample tasks and suggested solutions were developed and validated by classroom teachers

of mathematics, but **they have not been validated with students**. They represent models of the kinds of questions and problems students should be able to solve, and the kinds of activities students should be able to perform to meet the specific outcome to which they are linked.

The following two examples illustrate the components expected in a student's written response to a problem, when a graphing calculator has been used to assist in determining the answer.

Example 1: When asked to solve a problem that asks for the coordinates of key points, from a description of a situation that can be expressed as a function, the following should be included in the student's response:

- the function in $y =$ form
- a sketch of the graph with the scales labelled
- the key points identified on the sketch
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Example 2: When solving a problem that involves using the regression menu on a graphing calculator to determine the line of best fit for a data set, students can be expected to include the following in the written response:

- the data from input lists
- the regression formula used
- the equation of the line of best fit
- a sketch of the graph with the scales labelled
- a statement describing the relationship between the calculator screen output and the answer to the problem, within its context.

Descriptions of Student Performance (Related to Specific Outcome)

Each specific outcome is followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions represent the judgements of practising classroom teachers in regard to how their students would perform on the particular specific outcome.

Descriptions of Student Performance (Related to Sample Tasks)

The sample tasks for a particular specific outcome are followed by a description of student performance at the acceptable standard and at the standard of excellence. These descriptions, though **not based on actual student responses**, represent the judgements of practising classroom teachers in regard to how their students would perform, given these particular tasks. For many of the sample tasks, it was presumed that students would have access to reference materials, such as calculator manuals, software documentation and the reference utility materials present in their textbooks.

STANDARDS IN GRAPHING AND DESIGN

GENERAL OUTCOME:

- Analyze graphs or charts of given situations to derive specific information.

SPECIFIC OUTCOMES:

- 1.1 Extract information from given graphs of discrete or continuous data, using:
 - time series
 - glyphs (custom pictorial representations)
 - continuous data
 - contour lines.[C, CN, E, PS, R, V]
- 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data.
[CN, E, PS, V]
- 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

General and Specific Outcomes	
General Outcome	
Analyze graphs or charts of given situations to derive specific information.	
Specific Outcome	
1.1	Extract information from given graphs of discrete or continuous data, using: - time series - glyphs (custom pictorial representations) - continuous data - contour lines. [C, CN, E, PS, R, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Specific outcomes 1.1, 1.2 and 1.3 are closely related.
- Contour maps may be available from survey companies, geophysical data processing companies or oil companies, or may be purchased from map stores or through the Internet.
- Accommodation guides contain examples of glyph systems. Examples may be found at travel offices and travel agencies.
- Additional graphs can be obtained from the Statistics Canada web site at <<http://www.statcan.ca>>.
- The glyphs component of this outcome is addressed in the Addison-Wesley *Applied Mathematics 11 Student Project Book*.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: - extract information from a graphical presentation - extract information from a pictograph or glyph - use the information presented in a graph to perform simple mathematical calculations and to make interpretations.	A student demonstrating the <i>standard of excellence</i> can also: - design and explain the structure of multilevel glyphs - provide explanations for interpretations.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

1.1 Extract information from given graphs of discrete or continuous data, using:

- time series
- glyphs (custom pictorial representations)
- continuous data
- contour lines.

[C, CN, E, PS, R, V]

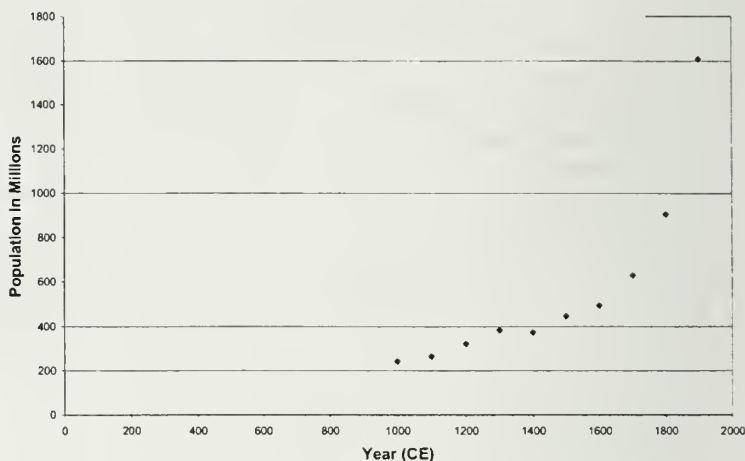
Sample Tasks

- | | |
|-----|-----------------|
| [✓] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

Question:

1. Study the following graph to answer the questions.

Approximate World Population (Millions)



- a. Are these data discrete or continuous? Explain.
- b. What was the estimated population of the world in 1700?
- c. What is the population trend between the years 1000 and 1400?
- d. Explain why there is no population data for the years prior to 1000.
- e. Account for the apparent decrease in population between the years 1300 and 1400.

Solution:

1. a. Continuous, because there are population values for all of the time periods in between.
- b. Approximately 630 000 000.
- c. The population is rising slowly.
- d. It is most likely that census data was not available prior to the year 1000.
- e. The most likely reason for the decrease would be the Black Death (bubonic plague). Other acceptable answers are such things as crop failure, war.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

1.1 Extract information from given graphs of discrete or continuous data, using:

- time series
- glyphs (custom pictorial representations)
- continuous data
- contour lines.

[C, CN, E, PS, R, V]

Sample Tasks

- | | | |
|-----|--|-----------------|
| [✓] | | Conceptual |
| | | Procedural |
| [✓] | | Problem-solving |

(continued)

Question:

2. A campground has the following symbols next to its listing in a camping guide.



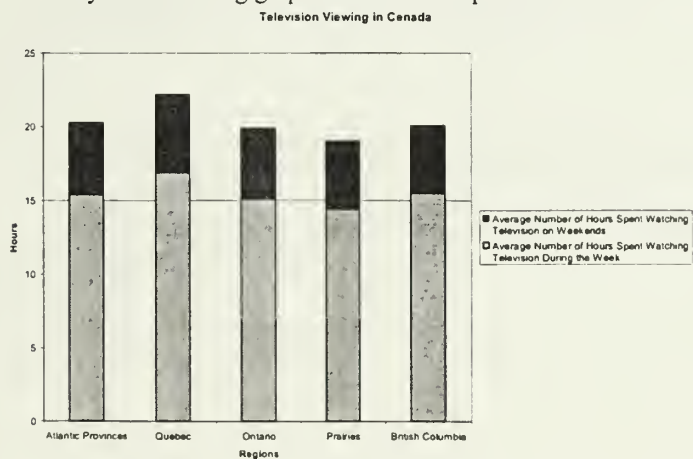
- Provide an interpretation for each of the 12 symbols.
- The owners of the campground will be adding laundry and shower facilities next year. Draw symbols that would clearly advertise these features.
- The owners also want to let campers know that open fires are not permitted at campsites, rather they are permitted only in designated areas. Design symbols to convey this information.

Solution:

- Reading from left to right, starting in the first row: Boat launch, Canoes for rental, Fishing, Food and drink, Hiking trail, Boating, Covered picnic tables, Children's playground, RV tank disposal, Swimming, Washrooms for disabled, Tent and RV spaces.
- Answers will vary.
- Answers will vary.

Question:

3. Study the following graph to answer the questions.



- On average, how many hours of television do viewers in the Prairies watch on weekends?
- On average, how many hours of television do viewers in British Columbia watch during a seven-day period?
- Estimate the average number of hours that a viewer in Ontario watches on a weekday; e.g., Tuesday.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

1.1 Extract information from given graphs of discrete or continuous data, using:

- time series
- glyphs (custom pictorial representations)
- continuous data
- contour lines.

[C, CN, E, PS, R, V]

Sample Tasks

- | | |
|-----|-----------------|
| [✓] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

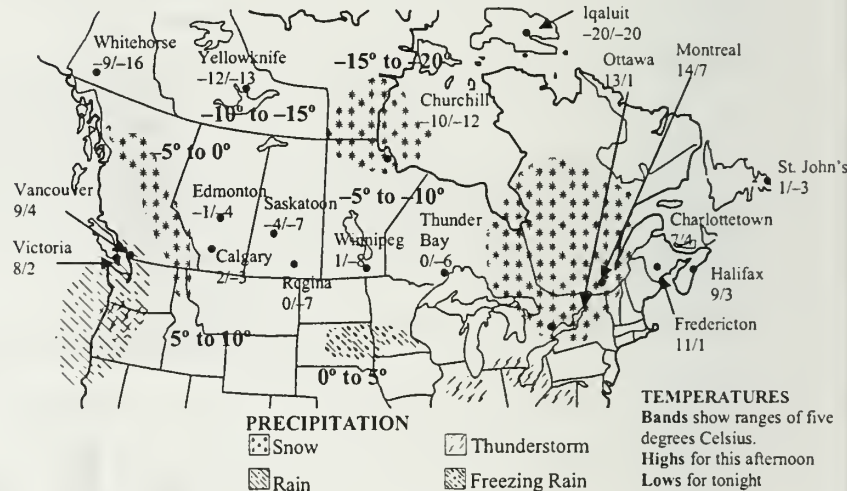
(continued)

Solution:

3. a. Approximately 4 hours.
- b. Just over 20 hours.
- c. Ontarians watch an average of 15 hours during the week, excluding weekends. Therefore, they watch approximately 3 hours per weekday.

Question:

4. Study the weather map to answer the questions below.
 - a. Identify which of the cities shown are going to experience nighttime temperatures between -6°C and -20°C .
 - b. Which cities are expecting both snow and an afternoon maximum temperature that is between 9°C and 14°C ?



Solution:

4. a. Whitehorse, Yellowknife, Saskatoon, Regina, Winnipeg, Churchill, Thunder Bay, Iqaluit.
- b. Montreal, Ottawa, Toronto.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

1.1 Extract information from given graphs of discrete or continuous data, using:

- time series
- glyphs (custom pictorial representations)
- continuous data
- contour lines.

[C, CN, E, PS, R, V]

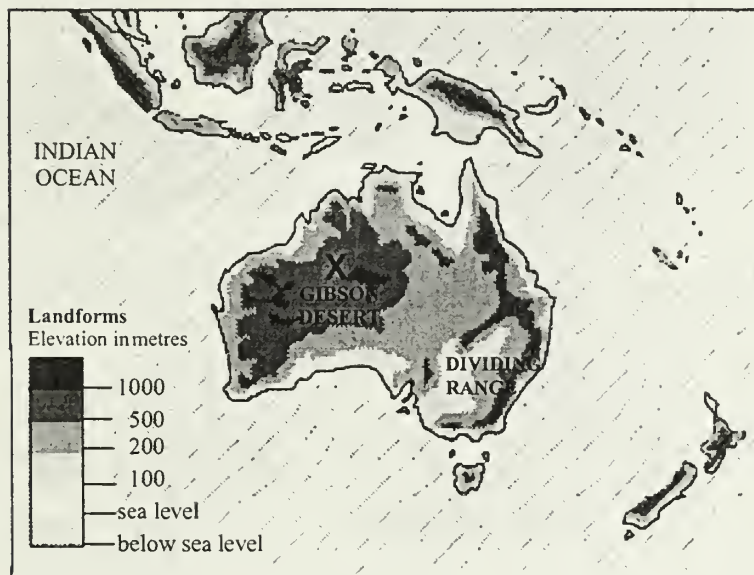
Sample Tasks

- | | |
|---|-----------------|
| ✓ | Conceptual |
| ✓ | Procedural |
| ✓ | Problem-solving |

(continued)

Question:

5. Given this elevation map of landforms in Australia, answer the following:



- What range of elevation is shown on this map of Australia?
- What is the elevation of point X in the Gibson Desert?
- Describe the elevations of the Dividing Range.

Solution:

5.
 - The range is from sea level to more than 1000 metres.
 - The elevation of point X in the Gibson Desert is between 500 m and 1000 m.
 - The Dividing Range includes elevations of 100, 200, 500 and 1000 metres.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

1.1 Extract information from given graphs of discrete or continuous data, using:

- time series
- glyphs (custom pictorial representations)
- continuous data
- contour lines.

[C, CN, E, PS, R, V]

Sample Tasks

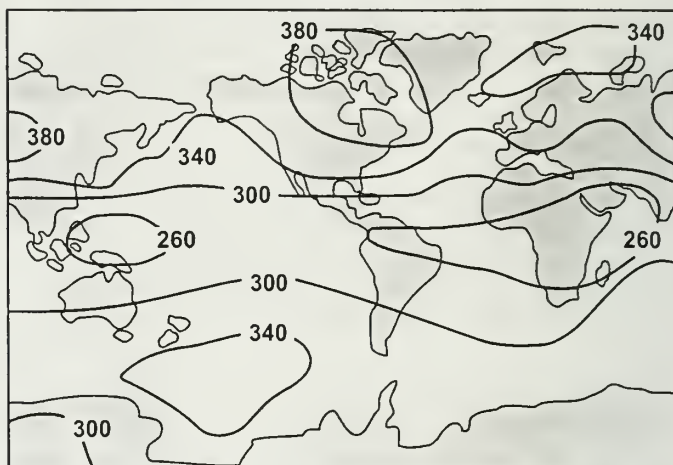
- | | |
|-----|-----------------|
| [✓] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

(continued)

Question:

6. Study this contour map and answer the questions.

Natural Ozone Levels Above the Earth



- What contour interval is used on this map?
- What is the ozone level over Alberta?
- Describe the two regions with the lowest levels of naturally occurring ozone.

Solution:

6. a. The interval is 40 units.
b. The ozone level over Alberta is between 340 and 380 units.
c. Region 1 lies above Indonesia and the Philippines. Region 2 lies largely over the lower part of Africa, the upper part of South America and the Atlantic Ocean in between.

(continued)

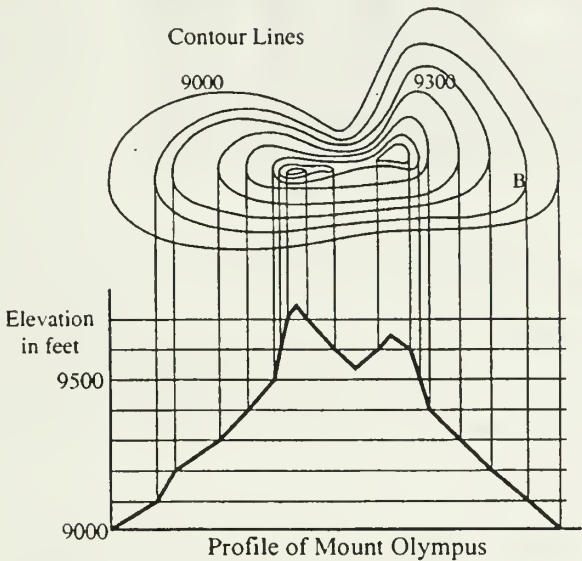
General and Specific Outcomes
General Outcome Analyze graphs or charts of given situations to derive specific information. Specific Outcome 1.1 Extract information from given graphs of discrete or continuous data, using: - time series - glyphs (custom pictorial representations) - continuous data - contour lines. [C, CN, E, PS, R, V]

Sample Tasks

✓	Conceptual
	Procedural
✓	Problem-solving

(continued)
Question:

7. Study the following contour drawing of Mount Olympus to answer the questions.



- Approximately how tall is the highest peak of Mount Olympus?
- What height label could be placed on the contour line B?
- Explain why some of the contour lines are close together while others are far apart.

Solution:

7. a. The height is approximately 9750 feet.
 b. B should be labelled 9100 feet.
 c. The close contour lines represent a steep slope while those further apart represent a gentler slope.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: read and analyze graphical presentations in questions 1, 2a, 3 to 6, 7a and 7b, and make interpretations of the information.	A student demonstrating the <i>standard of excellence</i> can also: - design suitable glyphs for question 2, parts b and c; however, this question is more closely related to specific outcome 1.3 - provide explanations for the contour lines in question 7, part c.

General and Specific Outcomes	
General Outcome	
Analyze graphs or charts of given situations to derive specific information.	
Specific Outcome	
1.2	Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The emphasis in this specific outcome is on reading and interpreting graphs and tables, not on collecting data and creating graphs.
- When interpolating and extrapolating from tabular data, students can use averages and/or midpoints.
- Specific outcomes 1.1, 1.2 and 1.3 are closely related.
- Additional data can be obtained from the Statistics Canada web site at <<http://www.statcan.ca>>.
- Population data lends itself to inferences regarding significant world events, such as World War II and the baby boom. Students will likely see links to social studies.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: • analyze and interpret trends in data presented in graphical and tabular form • interpolate or extrapolate, given a set of data in graphical or tabular form.	A student demonstrating the <i>standard of excellence</i> can also: • provide in-depth analysis and interpretations of trends in graphical and tabular data using information from other subject areas, where appropriate.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

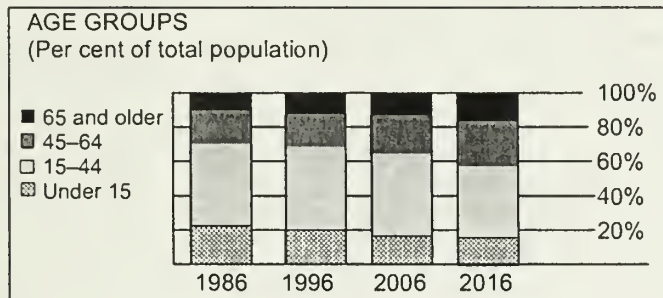
- 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

Sample Tasks

- | | |
|-----|-----------------|
| [✓] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

Question:

1.



- What is the overall trend for people 65 and older?
- In what year is the percentage of people in the 15–44 age group the smallest?
- What group tends to make up the largest percentage of the population in each year?
- What other trends do you see in the data?
- What other type of graph could be used to convey the same data?

Solution:

- The trend is that the percentage of people 65 and older is increasing.
 - In 2016, the percentage of people in the 15–44 age group will be smallest.
 - The 15–44 age group tends to make up the largest segment of the population.
 - Answers will vary, but students may say that the under 15 population is decreasing and the 45–64 age group is increasing.
 - Four separate pie charts could be drawn, one for each year, and then the four pie charts could be used to answer the above questions.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

Sample Tasks

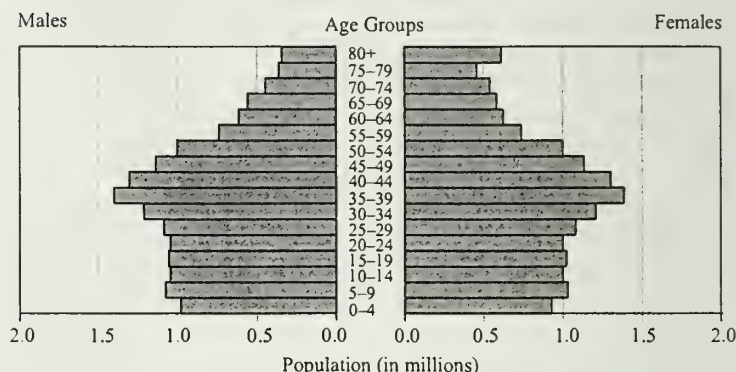
- [✓] Conceptual
[] Procedural
[✓] Problem-solving

(continued)

Question:

2. Study the graph of population data for Canada, and answer the questions that follow.

Canada's Population in 1998



- What is the ratio of males to females in the eighty and older age group? Make a hypothesis regarding the lack of symmetry in the graph for this age group.
- Approximately what per cent of the population was between 15 and 19 if the total population of Canada in 1998 was approximately 30 600 000?
- Estimate how many people were 65 or older in 1998.
- The baby boom was a period of time characterized by a greater number of births than in the years before and after. What evidence is there of a baby boom, and in what years did it likely occur?

Solution:

2. a. The ratio of males to females for the 80+ age group is $\frac{0.4 \text{ million}}{0.6 \text{ million}}$ or $\frac{2}{3}$.

Possible hypothesis: Women in Canada have a longer life span because they die less frequently of heart disease and cancer, perhaps because they were less likely to be exposed to environmental factors such as cigarette smoke or workplace hazardous materials in their earlier years.

- Per cent between 15 and 19

$$= \frac{1.1 \text{ million males} + 1.0 \text{ million females}}{30.6 \text{ million}} \times 100\% = 6.8\%$$
- In 1998 the number of people older than 65 was just under $(0.3 + 0.35 + 0.4 + 0.55 + 0.6 + 0.45 + 0.55 + 0.60)$ million = 4 million.
- The baby boom years would likely be between 1945 and 1960 because of the bulge in the population for those aged 35 to 50 who would have been born in that time period.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

Sample Tasks

- | | | |
|-----|--|-----------------|
| [] | | Conceptual |
| [✓] | | Procedural |
| [] | | Problem-solving |

(continued)

Question:

3. An amount of money is invested in an RRSP. The growth of the value is as follows:

Time (years)	Value
0	\$7 000
1	\$7 630
2	\$8 316
3	\$9 065
4	\$9 881
5	\$10 770

- What was the initial amount invested?
- Estimate when the investment will be worth \$8700. Justify your answer.
- Estimate what the RRSP will be worth after 6 years. Justify your answer.

Solution:

3. a. The initial investment was \$7000.
 b. The RRSP will reach \$8700 between 2 and 3 years. The actual answer can be justified using either ratio, per cent or midpoint.
 c. After 6 years the value will be approximately \$12 000, justified using ratio or per cent.

Question:

4. **Age Distribution of World Population**



(continued)

General and Specific Outcomes	
General Outcome	
Analyze graphs or charts of given situations to derive specific information.	
Specific Outcome	
1.2	Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

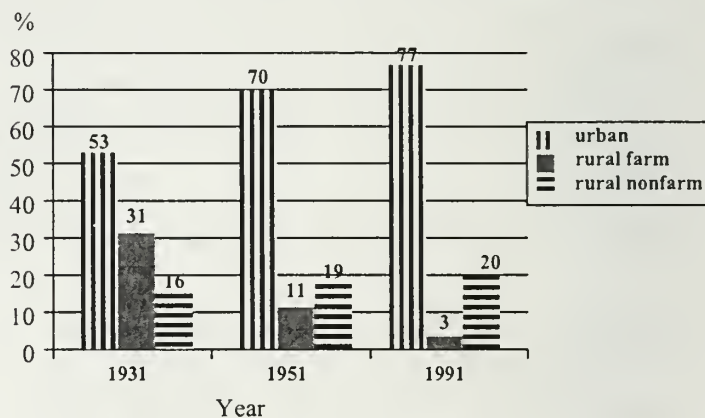
4. a. Use extrapolation and the graph to estimate when the world population of the age 60 or over group will equal the world population of the under age 15 group. Explain how you obtained your answer.
- b. Make an inference about the rise and fall of the population of the under age 15 group between 1950 and 1980.

Solution:

4. a. The populations will be estimated to be equal in the year 2050. This can be determined by extrapolating each line until the lines meet.
- b. The rise and fall of the population of the under age 15 group might be the result of an increased birth rate between 1935 and 1950 and a decreased birth rate between 1965 and 1980. This would be related to socioeconomic events happening on a worldwide basis; e.g., the mass movements of populations at the end of World War II or the implementation of birth control.

Question:

5. **Urban/Rural Population, Canada, 1931–1991 (in per cent)**



(continued)

General and Specific Outcomes
General Outcome Analyze graphs or charts of given situations to derive specific information. Specific Outcome 1.2 Draw and validate inferences, including interpolations and extrapolations, from graphical and tabular data. [CN, E, PS, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

5. a. Describe the trends in the graph above.
- b. Provide possible reasons for the trends.

Solution:

5. a. Trend 1: Increase in urban population over time.
Trend 2: Sharp decrease in rural farm population over time.
Trend 3: Moderate increase in rural nonfarm population over time.
- b. Between 1931 and 1991, many people decided that farming was not a viable way to make a living. They sold their farms and moved to urban centres, or settled in small towns or on acreages outside of urban areas. Urban areas may also have gained in population because of immigration of people from other countries.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating at the <i>acceptable standard</i> can: - provide solutions to questions 1 to 5 - provide simple justifications for the solutions to question 2, part a, question 4, part b, and question 5, part b, based only on the information provided.	A student demonstrating the <i>standard of excellence</i> can also: provide detailed justifications for the solutions to question 2, part a, question 4, part b, and question 5, part b, based on a broad range of knowledge.

General and Specific Outcomes
General Outcome Analyze graphs or charts of given situations to derive specific information. Specific Outcome 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- Specific outcomes 1.1, 1.2 and 1.3 are closely related.
- It is recommended that students start with simple cases that can be graphed by hand and progress to using technology for more complex cases.
- More complex graphical presentations; e.g., stacked bar charts, population pyramids or complex pie charts, can be generated using the graphing capabilities of a spreadsheet program or similar technology.
- Misleading graphs are available in abundance in the daily newspapers and business magazines.
- When analyzing graphical presentations for truthful display of data, it is important to consider both scale and previous trends.
- When using technology to solve problems, students should be expected to include the window setting and a sketch of the labelled graph in their solution.
- Some spreadsheet programs can be used to produce graphs with two scales.
- Additional data can be obtained from the Statistics Canada web site at <<http://www.statcan.ca>>.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: • determine if a graphical presentation is misleading and explain why.	A student demonstrating the <i>standard of excellence</i> can also: • discuss the advantages and disadvantages associated with different graphical presentations • provide alternative graphical presentations for data.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

Sample Tasks

- [✓] | Conceptual
- [✓] | Procedural
- [✓] | Problem-solving

Question:

1. Illegal Sales of Cigarettes

A total of 5023 retailers in 26 cities in all 10 provinces were tested by sending an underage teenager into the store to attempt to purchase cigarettes. The provincial breakdown for stores that broke the law is as follows:

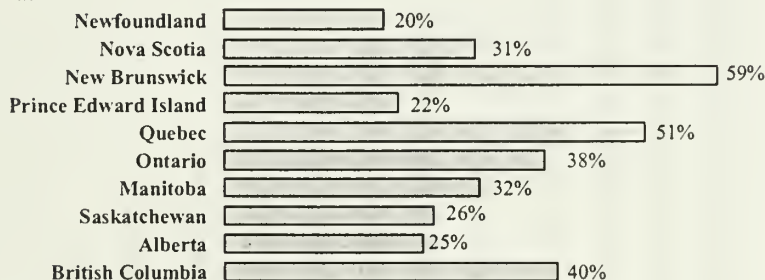
Province	Stores Breaking the Law (%)
Newfoundland	20
Nova Scotia	31
New Brunswick	59
Prince Edward Island	22
Quebec	51
Ontario	38
Manitoba	32
Saskatchewan	26
Alberta	25
British Columbia	40

- a. Design a bar graph to represent this data.
- b. Explain why a line graph or a circle graph would not be suitable for displaying this data.

Solution:

Illegal Sales of Cigarettes

1. a.



- b. The data illustrate a comparison and are not meant to show trends, for which a line graph is more suited. Also, the data are discrete. A circle graph is not suitable because each per cent does not represent a portion of the whole—100%.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

Sample Tasks

- [✓] | Conceptual
[] | Procedural
[✓] | Problem-solving

(continued)

Question:

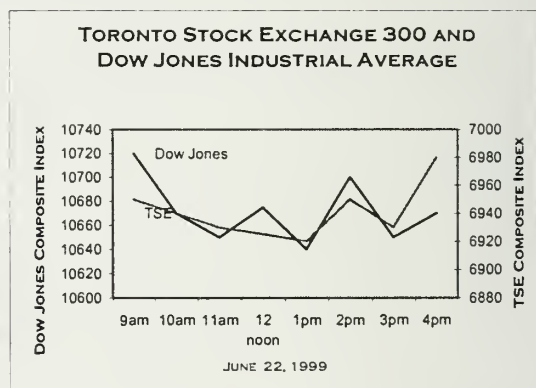
2. a. Making use of two vertical scales, graph the data from the following table.

Time	Dow Jones Industrial Average	Toronto Stock Exchange 300
9:00 am	10 720	6 950
10:00 am	10 670	6 940
11:00 am	10 650	6 930
12:00 noon	10 675	6 925
1:00 pm	10 640	6 920
2:00 pm	10 700	6 950
3:00 pm	10 650	6 930
4:00 pm	10 670	6 980

- b. Between 12:00 noon and 1:00 pm, which composite index experienced the greatest change and how much was it?
c. Between 11:00 am and 12:00 noon, which composite index experienced the greatest change and how much was it?
d. What is the change from the opening to the closing for the day for each composite index?
e. What is an advantage of using two vertical scales? What is a disadvantage?

Solution:

2. a.



- b. The Dow Jones Industrial Average changed by 35 points.
c. The Dow Jones Industrial Average experienced the larger change by increasing 25 points.
d. Over the day the Dow Jones dropped by 50 points. The TSE rose by 30 points.
e. Two vertical scales allow for comparison of patterns within the data but make it difficult to read detailed information.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

Sample Tasks

- [✓] | Conceptual
- [✓] | Procedural
- [✓] | Problem-solving

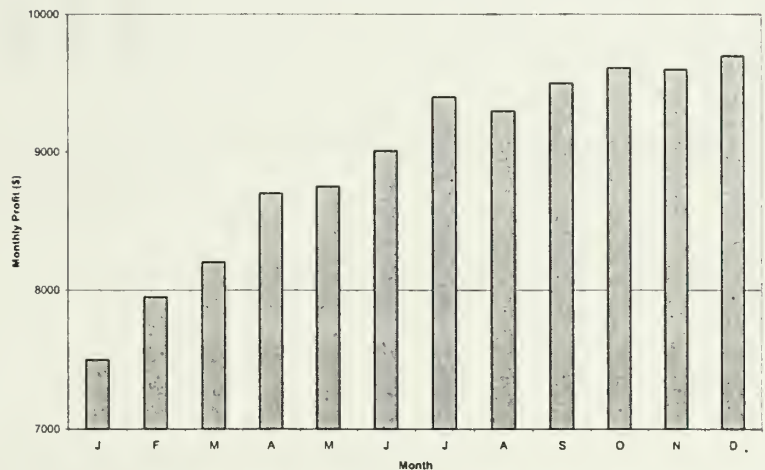
(continued)

Question:

3. The Acme Company, wanting to make a favourable impression on its shareholders, uses the following graph in its presentation. The graph is based on this data.

1998 Monthly Profits for Acme Company	
Month	Profit
January	\$7500
February	\$7950
March	\$8200
April	\$8700
May	\$8750
June	\$9010
July	\$9400
August	\$9300
September	\$9500
October	\$9610
November	\$9600
December	\$9700

1998 Monthly Profits for Acme Company



- How does the presentation of the data make the results appear more favourable?
- Create a graph that is a more truthful representation of the data.
- What other information would help a shareholder make a decision about further investments in this company?

(continued)

General and Specific Outcomes
General Outcome Analyze graphs or charts of given situations to derive specific information. Specific Outcome 1.3 Design different ways of presenting data and analyzing results, by focusing on the truthful display of data and the clarity of presentation. [C, CN, T, V]

Sample Tasks

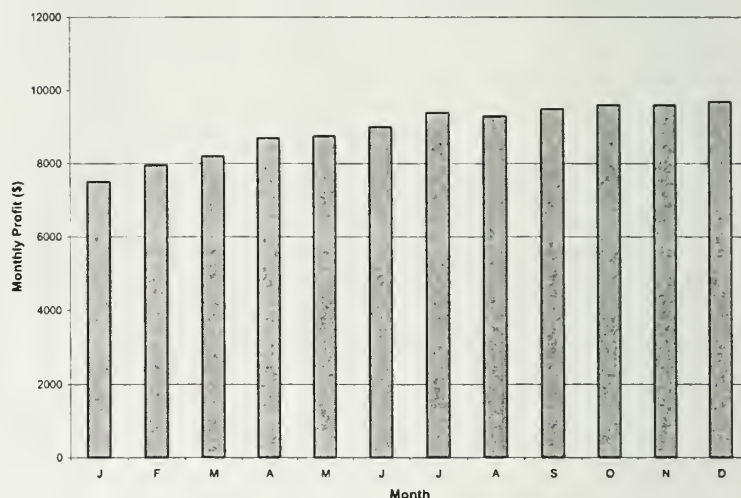
- [✓] | Conceptual
[✓] | Procedural
[✓] | Problem-solving

(continued)

Solution:

3. a. The graph may be misleading because it suggests that profits are rising more dramatically than they actually are. This is because of the scale chosen for the y-axis. If the origin was at (0, 0), the graph would be flatter.
- b. An alternative graph is given below.

1998 Profits for Acme Company



- c. Other useful information to shareholders would be data from previous years and data from similar companies.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: design and interpret graphs to represent the original data in questions 1, 2 and 3.	A student demonstrating the <i>standard of excellence</i> can also: - provide explanations as to why some data and graphical presentations can be misleading, as in question 3, part a - discuss and design alternative graphical presentations, as in question 1, part b and question 3, part b.

STANDARDS IN REGRESSION AND NONLINEAR EQUATIONS

GENERAL OUTCOMES:

- Represent and analyze situations that involve expressions, equations and inequalities.
- Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.
- Analyze graphs or charts of given situations to derive specific information.

SPECIFIC OUTCOMES:

- 2.1 Solve nonlinear equations, using a graphing tool. [CN, T, V]
- 2.2 Determine the following characteristics of the graph of a quadratic function:
- vertex
 - domain and range
 - axis of symmetry
 - intercepts.
- [C, PS, T, V]
- 2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:
- $y = ab^x$
 - $y = ax^2 + bx + c$.
- [C, CN, PS, T, V]
- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
- [C, CN, PS, T, V]
- 2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions of the form:
- $y = ab^x \rightarrow$ parameters a, b
 - $y = ax^2 + bx + c \rightarrow$ parameters a, c .
- [C, CN, R, V]

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

Strand: Patterns and Relations (Relations and Functions)

Students will:

- use algebraic and graphical models to generalize patterns, make predictions and solve problems.

Strand: Statistics and Probability (Data Analysis)

Students will:

- collect, display and analyze data to make predictions about a population.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

- 2.1 Solve nonlinear equations, using a graphing tool. [CN, T, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 1.2.
- The two graphical methods for solving an equation such as $x^3 + x = x^2 + 5$ are:
 1. graph $y_1 = x^3 + x$ and $y_2 = x^2 + 5$ and determine the x -coordinate(s) of the intersection point(s)
 2. graph $y = (x^3 + x) - (x^2 + 5)$ and find its zeros.
- The intent of this specific outcome is to solve nonlinear equations, but it does not extend to solving systems of nonlinear equations.
- Solving systems of nonlinear equations is not intended to be dealt with as its own independent topic.
- “Solve” in this outcome does not necessarily mean finding the exact roots of the equation. The focus should be on solving for a situation that can be expressed graphically.
- This specific outcome is linked to specific outcomes 2.2 and 2.3.
- This specific outcome is used throughout the course in a variety of contexts.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- solve a nonlinear equation graphically, using one method.

A student demonstrating the *standard of excellence* can also:

- solve a nonlinear equation graphically, using more than one method.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

2.1 Solve nonlinear equations, using a graphing tool. [CN, T, V]

Sample Tasks

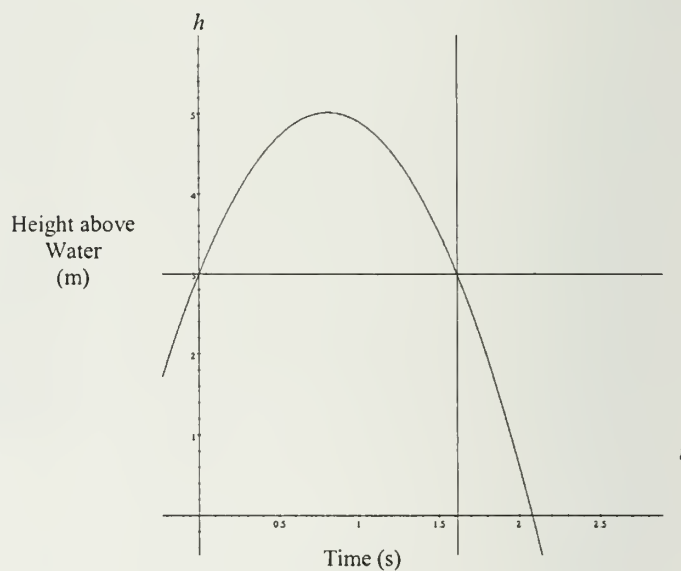
- | | |
|-----|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [✓] | Problem-solving |

Question:

1. The equation $h = -3.1t^2 + 5.0t + 3$ describes an Olympic diver diving off a 3-metre board.
 - a. Use the equation to predict how long after leaving the board a diver will enter the water.
 - b. Find the time the diver is above the diving board.

Solution:

Diving Off a 3-Metre Board



1. a. $h = 0$ when $t = 2.08$ s; the diver will reach the water after 2.08 s.
- b. The diver is above the board whenever $-3.1t^2 + 5.0t + 3 \geq 3$, or from $t = 0$ to $t = 1.61$ s, using the intersection points of $h = 3$ and $h = -3.1t^2 + 5.0t + 3$.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

- 2.1 Solve nonlinear equations, using a graphing tool. [CN, T, V]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

(continued)

Question:

2. Iodine-134 has a half-life of 52 minutes. A 100 mg sample of iodine-134 decays according to the following relation:
 $M = 100(2^{-t/52})$ where M is the mass in milligrams after time t in minutes.
- Rewrite the equation to represent the mass after a certain time as 15 milligrams.
 - How many minutes, to the nearest minute, would it take for 100 milligrams of iodine-134 to decay to 15 milligrams? Solve this graphically. Draw the image as it appears on your graphing calculator screen (label the axes appropriately), and fill in the blanks below.

$Y_1 =$
 $Y_2 =$
 $X \text{ min} =$
 $X \text{ max} =$
 $Y \text{ min} =$
 $Y \text{ max} =$

- Solve this problem using a different graphical method. Draw the image that appears on your graphing calculator screen (label the axes appropriately), and fill in the blanks below.

$Y_1 =$
 $Y_2 =$
 $X \text{ min} =$
 $X \text{ max} =$
 $Y \text{ min} =$
 $Y \text{ max} =$

(continued)

General and Specific Outcomes
General Outcome Represent and analyze situations that involve expressions, equations and inequalities.
Specific Outcome 2.1 Solve nonlinear equations, using a graphing tool. [CN, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:

2. a. $15 = 100(2^{-t/52})$
b. or c.

$$Y_1 = 100(2^{-x/52}) - 15$$

$X_{\min} = 0$
 $X_{\max} = 120$
 $Y_{\min} = -20$
 $Y_{\max} = 110$

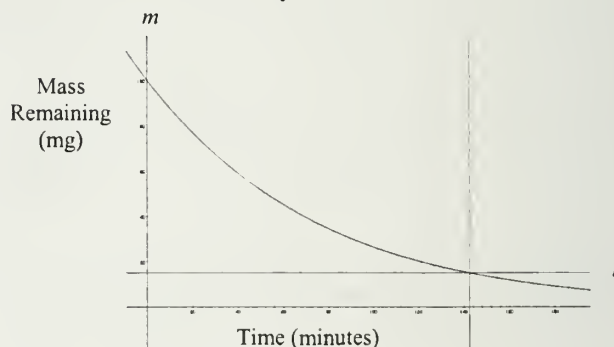
It would take 142 minutes for 100 milligrams of iodine-134 to decay to 15 milligrams.

$$Y_1 = 100(2^{-x/52})$$

$Y_2 = 15$
 $X_{\min} = 0$
 $X_{\max} = 120$
 $Y_{\min} = 0$
 $Y_{\max} = 110$

It would take 142 minutes for 100 milligrams of iodine-134 to decay to 15 milligrams.

Decay of Iodine-134



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: provide a solution to question 1, part a, and question 2, using a single method.	A student demonstrating the <i>standard of excellence</i> can also: provide a complete solution to question 1, part b, and to question 2, using alternative methods.

General and Specific Outcomes

General Outcome

Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.

Specific Outcome

2.2 Determine the following characteristics of the graph of a quadratic function:

- vertex
- domain and range
- axis of symmetry
- intercepts.

[C, PS, T, V]

[C] Communication

[CN] Connections

[E] Estimation and
Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 1.2.
- The intention of this specific outcome is to determine the characteristics of a quadratic function based on observation and interpretation of outputs from the graphing calculator.
- It is not intended that students use the process of completing the square to determine the characteristics of the quadratic function.
- Although not explicitly stated in the specific outcome, teachers should discuss direction of opening and minimum/maximum values.
- This specific outcome is linked to specific outcomes 2.1 and 2.3.
- It is the intent that students go beyond written statements for domain and range, but not go as formally as using set notation.
- The domain, range and intercepts may be restricted given the problem context.
- The vertex and intercepts should be expressed as ordered pairs.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- graph a quadratic function on a graphing calculator
- determine the following properties of a quadratic function, using a graphing calculator:
 - vertex
 - domain
 - range
 - equation of the axis of symmetry
 - x - and y -intercepts.

A student demonstrating the *standard of excellence* can also:

- link symmetry properties to the equation of a quadratic function
- distinguish between the domain, range and intercepts of the quadratic function and the problem context.

General and Specific Outcomes	
General Outcome	
Represent and analyze quadratic, polynomial and rational functions, using technology as appropriate.	
Specific Outcome	
2.2	Determine the following characteristics of the graph of a quadratic function: - vertex - domain and range - axis of symmetry - intercepts. [C, PS, T, V]

Sample Tasks

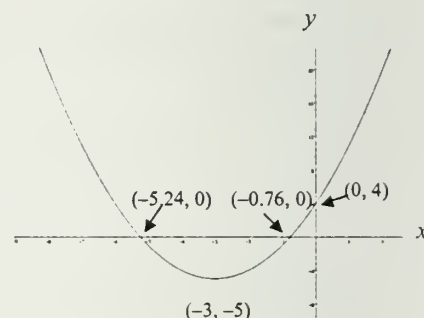
[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

Question:

- Graph $f(x) = x^2 + 6x + 4$ and determine the:
 - vertex
 - domain
 - range
 - equation of the axis of symmetry
 - x- and y-intercepts.

Solution:

- $y = x^2 + 6x + 4$
 - $(-3, -5)$
 - $x \in R$
 - $y \geq -5$
 - $x = -3$
 - x-intercepts $(-5.24, 0)$
 $(-0.76, 0)$
y-intercept $(0, 4)$



Question:

- If all you know about a quadratic function is that its x-intercepts are $(-5, 0)$ and $(3, 0)$, explain how you would determine the equation for the axis of symmetry.

Solution:

- The axis of symmetry is a vertical line passing through the vertex and vertically slicing the graph into two symmetrical parts. Therefore, the distance from one x-intercept to the axis of symmetry must be the same as the distance from the other x-intercept to the axis of symmetry. Find the midpoint of the segment between the intercepts, and write the equation of the vertical line running through the midpoint.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: complete all of question 1.	A student demonstrating the <i>standard of excellence</i> can also: provide a complete solution for question 2.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
- $y = ax^2 + bx + c$.

[C, CN, PS, T, V]

Notes:

- Students worked with linear regression models in Applied Mathematics 10; the intent for Applied Mathematics 20 is to examine quadratic and exponential regressions. Data sets should be limited to one of these three regression models.
- Students must consider more than the fit when deciding on the best regression model. They need to consider the trend of the data—whether it will continue to decrease, increase or change—before finalizing the regression model.
- Students will be more successful choosing regression models if they have a clear understanding of the characteristics of linear, quadratic and exponential functions, related to specific contexts.
- Students use best-fit graphs and curve fitting in their science studies.
- This specific outcome is intended to be addressed using the regression functions found on graphing calculators or within computer programs.
- Students can be guided to determine the equation of a line for simple cases, without the use of technology, and then validate the equation using technology.
- There should be a balance between collecting data for analysis and analyzing given data in this specific outcome.
- This specific outcome is linked to specific outcomes 2.1 and 2.2.
- Additional data sets can be obtained from the Internet.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- collect and analyze experimental data
- use a graphing calculator to create a scatterplot to determine the exponential regression equation and to determine the quadratic regression equation
- use technology to interpolate and extrapolate, based on the regression equation.

A student demonstrating the *standard of excellence* can also:

- choose the appropriate regression equation for a data set, based on the context and the correlation coefficient
- explain the choice of regression model used in a particular context.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
- $y = ax^2 + bx + c$.

[C, CN, PS, T, V]

Sample Tasks

- | | | |
|-----|--|-----------------|
| [✓] | | Conceptual |
| [✓] | | Procedural |
| [✓] | | Problem-solving |

Question:

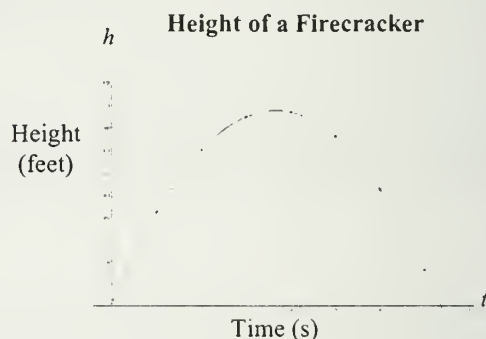
1. A firecracker is shot upward into the sky, producing the following data.

Time Elapsed (seconds)	Height of Firecracker (feet)
1	106
2	175
3	212
4	217
5	190
6	131
7	40

- a. Use a graphing calculator to draw a graph that illustrates the time elapsed versus the height of the firecracker.
- b. Use a quadratic regression model to determine the equation that best fits the data.

Solution:

1. a.



- b. $y = -16x^2 + 117x + 5$

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
 - $y = ax^2 + bx + c$.
- [C, CN, PS, T, V]

Sample Tasks

- | | |
|---|-----------------|
| ✓ | Conceptual |
| ✓ | Procedural |
| ✓ | Problem-solving |

(continued)

Question:

2. The following data represents the distance an object free falls over a certain time period.

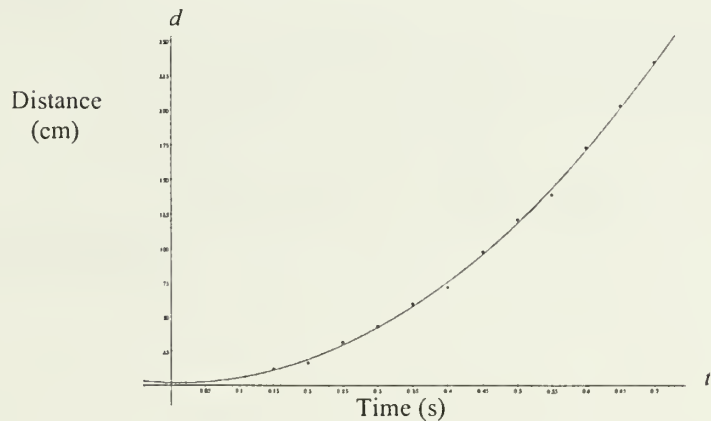
Time (seconds)	Distance (centimetres)
0.15	11.8
0.20	16.4
0.25	31.7
0.30	42.9
0.35	59.7
0.40	71.7
0.45	97.3
0.50	120.9
0.55	138.9
0.60	171.8
0.65	201.6
0.70	233.4

- Choose an appropriate regression model, and explain how you decided on this model.
- Find the equation of the curve of best fit. The coefficients should be to the nearest integer value.

Solution:

2. a. It appears to be a quadratic regression, because the data seems to increase dramatically.

Free Fall of an Object



- b. $y = 489x^2 - 12x + 2$, where y is the distance in centimetres and x is the time in seconds.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
- $y = ax^2 + bx + c$.

[C, CN, PS, T, V]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

(continued)

Question:

3. Collect data on a particular model of car to complete the table. Provide the source of your information.

Age of Car (years)	Price
1	
2	
3	
5	
6	
7	
9	

- Create a scatterplot of data, using your graphing calculator. Use x to represent the age of the car and y to represent the price.
- Determine the exponential regression equation that best represents this data, and determine its correlation coefficient.
- Determine the quadratic regression equation that best represents this data, and determine its correlation coefficient.
- Which equation best represents the data? Justify your answer.

Solution:

3. Sample data table:

Age of Car (years)	Price
1	\$18 495
2	\$16 995
3	\$10 700
5	\$6 100
6	\$5 600
7	\$5 000
9	\$3 375

The data relate to a particular model of mid-size car in the Calgary area. (Students should be more specific as to the model surveyed.)

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

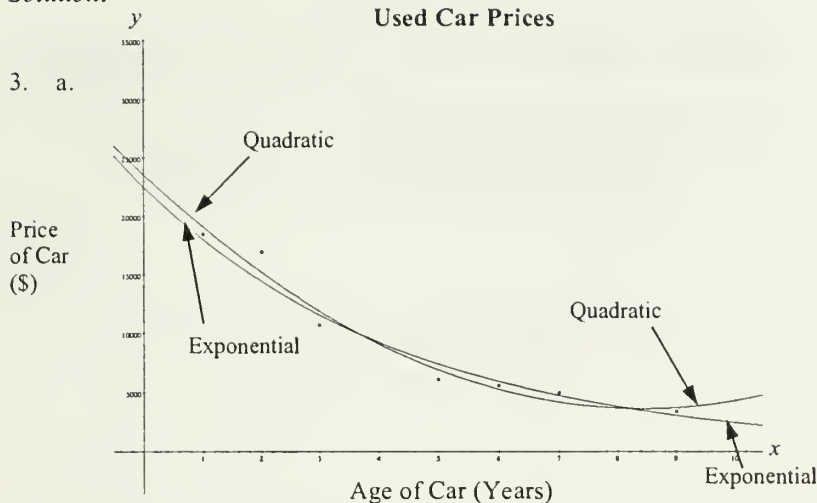
- $y = ab^x$
 - $y = ax^2 + bx + c$.
- [C, CN, PS, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:



b. $y = (22\,512)(0.801)^x$ $R^2 = 0.95233$

c. $y = 280.55x^2 - 4728.41x + 23\,541.33$ $R^2 = 0.95685$

- d. If you look at the correlation coefficient, the quadratic function is the best. However, in reality the quadratic function will eventually curve upward again. The price of the car will not increase. So the best function is likely to be the exponential.

Question:

4. a. Take a sheet of paper and tear or cut it successively into halves four times—each time placing the halves on top of each other before cutting again—to complete the first five lines of this table. After making 4 cuts, make a prediction about the pattern and complete the last two lines in the table.

Number of Cuts	Number of Pieces
0	
1	
2	
3	
4	
5	
6	

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
 - $y = ax^2 + bx + c$.
- [C, CN, PS, T, V]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

(continued)

Question:

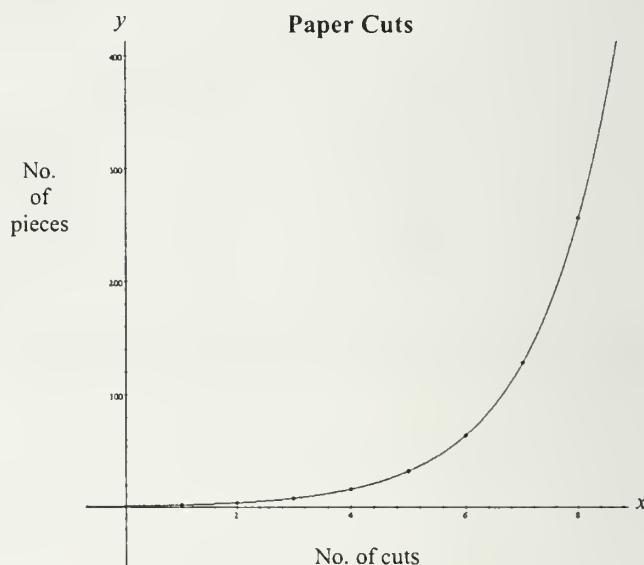
4. b. Use regression to determine the equation for this relationship. Sketch the graph. Use x to represent the number of cuts and y to represent the number of pieces.

Solution:

4. a.

Number of Cuts	Number of Pieces
0	1
1	2
2	4
3	8
4	16
5	32
6	64

- b. The equation for this relationship is $y = 2^x$.



(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.3 Collect experimental data; graph the data using technology and represent the data with best-fit exponential or quadratic functions of the form:

- $y = ab^x$
 - $y = ax^2 + bx + c$.
- [C, CN, PS, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

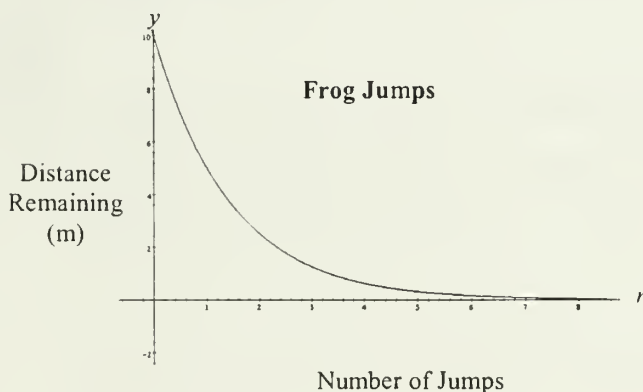
5. Imagine the following. A frog is jumping toward a pond. Each time the frog jumps(n) it lands at a point that is half the distance remaining to reach the pond (y).
- Use an investigation to collect data that simulates this scenario.
 - Determine the equation for this relationship, and sketch the graph you see in the window of your graphing calculator.

Solution:

5. a.

Number of Jumps	Distance Remaining (m)
0	10.00
1	5.00
2	2.50
3	1.25
4	0.63
5	0.31

- b. $y = 10\left(\frac{1}{2}\right)^n$ where n is the number of jumps.



Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- determine the regression equations, choose appropriate window settings and sketch the graphs for questions 1, 2, 3 and 4
- complete question 5, part b, after being provided with data for question 5, part a.

A student demonstrating the *standard of excellence* can also:

- explain the choice of regression models in question 2, part a, and question 3, part d
- complete the investigation needed to collect data for question 5, part a
- determine the appropriate equation and sketch the graph for question 5, part b.

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

General and Specific Outcomes	
General Outcome	
Analyze graphs or charts of given situations to derive specific information.	
Specific Outcome	
2.4	Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

Notes:

- Students worked with linear regression models in Applied Mathematics 10; the intent for Applied Mathematics 20 is to examine quadratic and exponential regressions. Data sets should be limited to one of these three regression models.
- Students must consider more than the fit when deciding on the best regression model. They need to consider the trend of the data—whether it will continue to decrease, increase or change—before finalizing the regression model.
- Students will be more successful choosing regression models if they have a clear understanding of the characteristics of linear, quadratic and exponential functions, related to specific contexts.
- Students use best-fit graphs and curve fitting in their science studies.
- This specific outcome is intended to be addressed using the regression functions found on graphing calculators or within computer programs.
- Students can be guided to determine the equation of a line for simple cases, without the use of technology, and then validate the equation using technology.
- This specific outcome is linked to specific outcomes 2.3 and 2.5.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: • analyze experimental data • use technology to interpolate and extrapolate, based on the regression equation • make predictions using interpolation and extrapolation • evaluate predictions made by interpolation.	A student demonstrating the <i>standard of excellence</i> can also: • explain the choice of regression model used in a particular context • evaluate predictions made by extrapolation.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
[C, CN, PS, T, V]

Sample Tasks

- [✓] | Conceptual
[✓] | Procedural
[✓] | Problem-solving

Question:

1. A firecracker is shot upward into the sky, producing the following data.

Time Elapsed (seconds)	Height of Firecracker (feet)
1	106
2	175
3	212
4	217
5	190
6	131
7	40

The data is best represented by the quadratic function

$$h(t) = -16t^2 + 117t + 5.$$

- What is the height of the firecracker after 4.5 seconds?
- How much time elapses before the firecracker begins to fall?
- How much time will have elapsed when the firecracker hits the ground?
- From what height was the firecracker fired?
- Explain why it would not be reasonable to extend the graph to quadrants 2, 3 and 4.

Solution:

- The height is approximately 207 feet, obtained by substituting $t = 4.5$ into the expression for $h(t)$.
 - It starts to fall after approximately 3.65 seconds, obtained as the t -coordinate of the maximum of $h(t)$.
 - It will hit the ground after approximately 7.49 seconds, obtained as the t -coordinate of the zero of $h(t)$.
 - The firecracker was fired from 5 feet above the ground, obtained by substituting $t = 0$ into the expression for $h(t)$.
 - Since it is not reasonable to deal with negative values of time or heights below ground, the graph should not be extended into quadrants that illustrate these possibilities.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
[C, CN, PS, T, V]

Sample Tasks

- [✓] | Conceptual
[✓] | Procedural
[✓] | Problem-solving

(continued)

Question:

2. The following data represents the distance an object free falls over a certain time period.

Time (seconds)	Distance (centimetres)
0.15	11.8
0.20	16.4
0.25	31.7
0.30	42.9
0.35	59.7
0.40	71.7
0.45	97.3
0.50	120.9
0.55	138.9
0.60	171.8
0.65	201.6
0.70	233.4

The data is best represented by the quadratic function $d(t) = 489t^2 - 12t + 2$, where t is the time in seconds, and d the distance in centimetres.

Use this equation to predict what the distance will be at a time of 0.95 seconds.

Solution:

2. The distance will be 432 cm when the time is 0.95 seconds.

$$y = 489(0.95)^2 - 12(0.95) + 2 = 432.1 \text{ cm.}$$

OR

Use the graphing calculator and the calculate or table function to determine that the distance is 432 cm.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
[C, CN, PS, T, V]

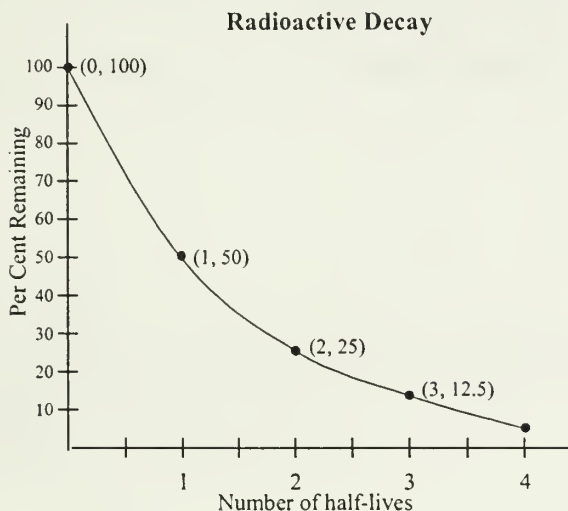
Sample Tasks

- [✓] | Conceptual
- [✓] | Procedural
- [✓] | Problem-solving

(continued)

Question:

3. The following measurements were taken for the amount of a radioactive substance remaining after a number of half-lives had passed. Use this information to answer the following questions.



- a. What percentage of the substance remains after 2 half-lives?
- b. How many half-lives are required for less than 2% to remain?
- c. If one half-life of this substance represents 8 minutes, how long would it take for only 10% of the substance to remain?
- d. This data, when the points of measurement are plotted, looks like discrete data. Explain how you know that this data is actually continuous.

Solution:

3. a. 25% remains after 2 half-lives.
b. After 6 half-lives, only 1.56% remains.
c. After 3.34 half-lives, or 26.7 minutes, only 10% would remain.
d. The decay does not occur in set increments. The decay is continuous.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
[C, CN, PS, T, V]

Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [✓] | Problem-solving |

(continued)

Question:

4. The following table represents the value of a particular model of mid-size car in the Calgary market.

Age of Car (years)	Price
1	\$18 495
2	\$16 995
3	\$10 700
5	\$6 100
6	\$5 600
7	\$5 000
9	\$3 375

The data can be represented either by the quadratic expression $P(t) = 280.5t^2 - 4728t + 23541$ or by the exponential expression $P(t) = (22\ 512)(0.801)^t$.

- Use both equations to determine the value of a new car.
- Use both equations to determine the value of this same model of car after 10 years.
- Which equation best represents the data? Justify your answer.

Solution:

- Exponential: \$22 512
Quadratic: \$23 541 for estimated price of a new car
 - Exponential: \$2 447
Quadratic: \$4 312 for estimated price of a 10-year-old car
- The quadratic function appears to be the best. However, in reality the quadratic function will eventually curve upward again. The price of the car will not increase. So the best function is likely to be the exponential.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems.
[C, CN, PS, T, V]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

(continued)

Question:

5. Mary took a sheet of paper and tore it successively into halves four times—each time placing the halves on top of each other before tearing again—to complete the first five lines of this table. She counted the pieces and recorded her results in the table.

Number of Cuts	Number of Pieces
0	1
1	2
2	4
3	8
4	16
5	32
6	64

- Use regression to determine the equation for this relationship. Sketch the graph. Use x to represent the number of cuts and y to represent the number of pieces.
- What is the minimum number of cuts that would have to be made to result in 1000 pieces of paper?

(continued)

General and Specific Outcomes
General Outcome Analyze graphs or charts of given situations to derive specific information.
Specific Outcome 2.4 Use best-fit exponential and quadratic functions and their associated graphs to make predictions and solve problems. [C, CN, PS, T, V]

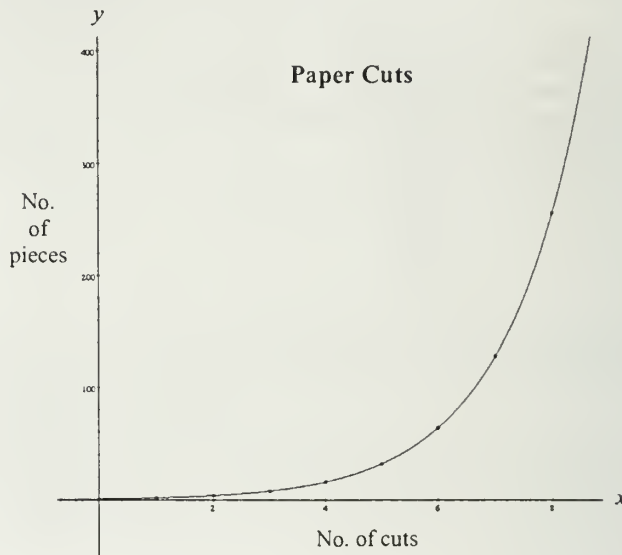
Sample Tasks

☐	Conceptual
☒	Procedural
☐	Problem-solving

(continued)

Solution:

5. a. The equation for this relationship is $y = 2^x$.



- b. It would require at least 10 cuts to obtain 1000 pieces.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: - complete the predictions required in question 1, parts a, b, c and d - answer question 2 completely - complete the predictions required in question 3, parts a, b and c - complete the predictions required in question 4, parts a and b - complete question 5, part a.	A student demonstrating the <i>standard of excellence</i> can also: - explain the domain and range in question 1, part e - explain the choice of regression models in question 4, part c - complete question 5, part b.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions of the form:
- $y = ab^x \rightarrow$ parameters a, b
 - $y = ax^2 + bx + c \rightarrow$ parameters a, c .
- [C, CN, R, V]

Notes:

- Students worked with linear regression models in Applied Mathematics 10; the intent for Applied Mathematics 20 is to examine quadratic and exponential regressions. Data sets should be limited to one of these three regression models.
- Students will be more successful choosing regression models if they have a clear understanding of the characteristics of linear, quadratic and exponential functions, related to specific contexts.
- Students use best-fit graphs and curve fitting in their science studies.
- This specific outcome is intended to be addressed using the regression functions found on graphing calculators or within computer programs.
- This specific outcome is linked to specific outcomes 2.3 and 2.4.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- identify meanings for positive values of a and b in an exponential function
- identify meanings for parameters a and c in quadratic functions.

A student demonstrating the *standard of excellence* can also:

- choose the appropriate regression equation for a data set, based on the context and the values of the parameters
- explain the choice of regression model used in a particular context
- identify meanings for negative values of b in an exponential function.

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions of the form:

- $y = ab^x \rightarrow$ parameters a, b
- $y = ax^2 + bx + c \rightarrow$ parameters a, b, c .

[C, CN, R, V]

Sample Tasks

- [✓] | Conceptual
- [✓] | Procedural
- [✓] | Problem-solving

Question:

1. A firecracker is shot upward into the sky, producing the following data.

Time Elapsed (seconds)	Height of Firecracker (feet)
1	106
2	175
3	212
4	217
5	190
6	131
7	40

The data is best represented by the quadratic function

$$h(t) = -16t^2 + 117t + 5.$$

What is the meaning of the 5 in the equation $h(t) = -16t^2 + 117t + 5$?

Solution:

1. The firecracker was fired from 5 feet above the ground.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions of the form:
- $y = ab^x \rightarrow$ parameters a, b
 - $y = ax^2 + bx + c \rightarrow$ parameters a, c .
- [C, CN, R, V]

Sample Tasks

- | | | |
|-----|--|-----------------|
| [✓] | | Conceptual |
| [✓] | | Procedural |
| [✓] | | Problem-solving |

(continued)

Question:

2. The following data represents the distance an object free falls over a certain time period.

Time (seconds)	Distance (centimetres)
0.15	11.8
0.20	16.4
0.25	31.7
0.30	42.9
0.35	59.7
0.40	71.7
0.45	97.3
0.50	120.9
0.55	138.9
0.60	171.8
0.65	201.6
0.70	233.4

The data is best represented by the quadratic function $d(t) = 489t^2 - 12t + 2$, where t is the time in seconds, and d the distance in centimetres.

What does the value 2 represent?

Solution:

2. The 2 represents the starting position.

(continued)

General and Specific Outcomes

General Outcome

Analyze graphs or charts of given situations to derive specific information.

Specific Outcome

- 2.5 Explain the significance of the parameters in the equations for exponential and quadratic functions of the form:
- $y = ab^x \rightarrow$ parameters a, b
 - $y = ax^2 + bx + c \rightarrow$ parameters a, c .
- [C, CN, R, V]

Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [✓] | Problem-solving |

(continued)

Question:

3. The following table represents the value of a particular model of mid-size car in the Calgary market.

Age of Car (years)	Price
1	\$18 495
2	\$16 995
3	\$10 700
5	\$6 100
6	\$5 600
7	\$5 000
9	\$3 375

The data can be represented either by the quadratic expression $P(t) = 280.5t^2 - 4728t + 23541$ or by the exponential expression $P(t) = (22\,512)(0.801)^t$.

- Use both equations to determine the value of a new car.
- Explain how the value was determined.
- What is the annual percentage depreciation rate? Justify your answer.

Solution:

3. The data relate to a particular model of mid-size car in the Calgary area. (Students should be more specific as to the model surveyed.)
- Exponential: \$22 512 for estimated price of a new car.
Quadratic: \$23 541 for estimated price of a new car.
 - The new car price is the value of the parameter a for the exponential expression and the parameter c for the quadratic expression.
 - The quadratic function appears to be the best. However, in reality, the quadratic function will eventually curve upward again. The price of the car will not increase. So the best function is likely to be the exponential, with $0.801 = 1 + i$ or $i = -19.9\%$ per year.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- answer questions 1 and 2 completely
- complete the predictions required in question 3, part a
- relate new price to parameters in question 3, part b.

A student demonstrating the *standard of excellence* can also:

- explain the choice of regression models in question 3, part c
- determine the value of the depreciation rate in question 3, part c.

STANDARDS IN LINEAR SYSTEMS AND PROGRAMMING

GENERAL OUTCOMES:

- Represent and analyze situations that involve expressions, equations and inequalities.
- Use linear programming to solve optimization problems.

SPECIFIC OUTCOMES:

- 3.1 Graph linear inequalities in two variables, including the conversion of $Ax + By + C = 0$ into $y =$ form. [PS, V]
- 3.2 Solve systems of linear equations, in two variables:
- algebraically (elimination and substitution)
 - graphically.
- [CN, PS, T, V]
- 3.3 Use expressions containing variables to describe problem contexts and solutions. [C, CN, PS, R]
- 3.4 Solve, graphically, systems of linear inequalities, in two variables, using technology. [CN, PS, T, V]
- 3.5 Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]
- 3.6 Apply linear programming to find optimal solutions to decision-making problems. [C, PS, R, T, V]

Strand: Patterns and Relations (Variables and Equations)

Students will:

- represent algebraic expressions in multiple ways.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

- 3.1 Graph linear inequalities in two variables, including the conversion of $Ax + By + C = 0$ into $y =$ form. [PS, V]

[C] Communication

[CN] Connections

[E] Estimation and

Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 1.1.
- Students were introduced to inequalities with one variable in Grade 9 mathematics, where they solved inequalities and graphed inequalities on a number line.
- Coordinate geometry skills are essential for this specific outcome. The concepts of plotting points and intercepts, line graphing, and the use of calculators are the more important concepts to review.
- $Ax + By + C = 0$ can be sketched using intercepts.
- Conversion from the $Ax + By + C = 0$ form to any $y =$ form is a necessary preliminary step to the use of a graphing calculator.
- Window settings, on the graphing calculator, are useful to replace the plotting of horizontal and vertical lines. For example, $x < 5$ could be entered into window settings as an x -max of 5.
- Students should first graph manually—to solve for and sketch the solution region—and then graph using the graphing calculator. An even balance between both approaches is recommended.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- graph the boundary line between two half planes
- use a test point, usually $(0, 0)$, to determine the solution region that satisfies the inequality, given a boundary line
- graph a linear inequality expressed in the form $y = mx + b$, using $<$, $>$, $\leq$, $\geq$
- rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, where A , B , C are integral and $B > 0$.

A student demonstrating the *standard of excellence* can also:

- distinguish between the use of solid and broken lines in solution regions
- graph any linear inequality in two variables
- rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, and graph
- explain why the shaded half plane represents the solution region of the inequality.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

- 3.1 Graph linear inequalities in two variables, including the conversion of $Ax + By + C = 0$ into $y =$ form. [PS, V]

Sample Tasks

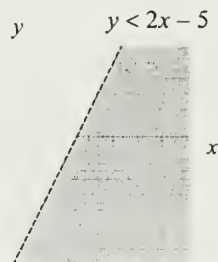
		Conceptual
	✓	Procedural
		Problem-solving

Question:

1. Graph $y < 2x - 5$.

Solution:

1.

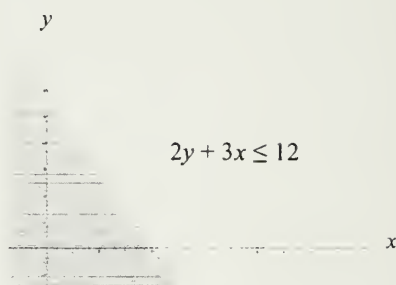


Question:

2. Graph $2y + 3x \leq 12$.

Solution:

2.



(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

- 3.1 Graph linear inequalities in two variables, including the conversion of $Ax + By + C = 0$ into $y =$ form. [PS, V]

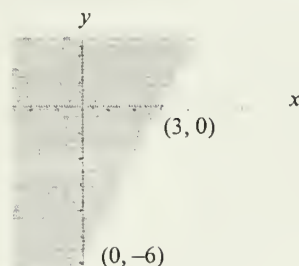
Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

3. Given the following graph, provide the inequality.



Solution:

$$3. \quad \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{0 - (-6)}{3 - 0} = 2$$

$$y\text{-intercept} = -6$$

$$y \geq 2x - 6.$$

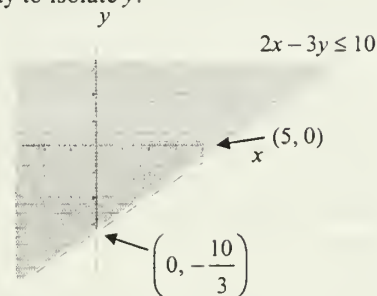
Question:

4. Graph $2x - 3y \leq 10$ with a graphing calculator.

Solution:

4. Rewrite the inequality to isolate y .

$$y \geq \frac{2}{3}x - \frac{10}{3}$$



Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- complete questions 1 and 2 on graph paper or on a graphing calculator, but may use a solid line rather than a broken line
- determine the x -intercept as $(5, 0)$ in question 4.

A student demonstrating the *standard of excellence* can also:

- complete questions 1, 2 and 3, including the boundaries
- complete question 4.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Solve systems of linear equations, in two variables:

- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 1.2.
- Students will need to review manipulation of an equation and simplifying expressions.
- Students will need to be familiar with functions on the graphing calculator, including zoom and windows.
- Using reasoning, students should choose the most efficient method to solve a system.
- Graphing involves a balance of paper-and-pencil and calculator graphing.
- Students should know the intersection command on the calculation function.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- graphically solve systems of equations that involve direct substitution without manipulation into a $y =$ form
- graphically solve systems of equations with integral coefficients with manipulation into a $y =$ form
- determine the point of intersection
- indicate that the point of intersection is a solution to the system
- translate word problems into appropriate systems of equations when the information is contained within the scope of the context or based on common mathematical knowledge
- solve linear systems of equations algebraically by substitution, where at least one variable in one equation has a coefficient of one
- solve linear systems of equations with integral coefficients by elimination
- recognize when it is more efficient to use substitution.

A student demonstrating the *standard of excellence* can also:

- graphically solve systems of equations with nonintegral coefficients with manipulation into a $y =$ form
- translate word problems into a system of equations, when additional background information is required to understand the context
- solve linear systems of equations where some or all coefficients are nonintegral
- recognize when a system has no point of intersection.

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Solve systems of linear equations, in two variables:

- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

		Conceptual
	✓	Procedural
		Problem-solving

Question:

1. Solve the system algebraically and graphically.

$$y = 5x - 9 \quad \textcircled{1}$$

$$3x + 4y = 10 \quad \textcircled{2}$$

Solution:

Algebraic

1. In $\textcircled{2}$ let $y = 5x - 9$

$$3x + 4(5x - 9) = 10$$

$$23x - 36 = 10$$

$$23x = 46$$

$$x = 2$$

In $\textcircled{1}$ let $x = 2$

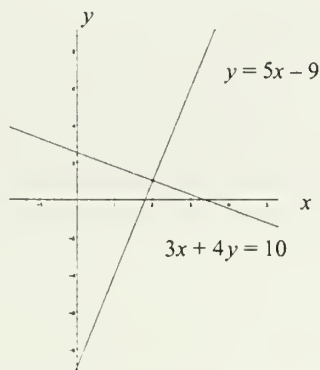
$$y = 5(2) - 9$$

$$y = 1$$

Therefore, the solution is

$$x = 2, y = 1.$$

Graphical



Question:

2. Solve the system algebraically and graphically.

$$5y - 3x = 30 \quad \textcircled{1}$$

$$2y - 5x = 50 \quad \textcircled{2}$$

Solution:

2. Algebraic

Multiply $\textcircled{1}$ by 2 and $\textcircled{2}$ by 5.

$$10y - 6x = 60 \quad \textcircled{3}$$

$$10y - 25x = 250 \quad \textcircled{4}$$

Subtract $\textcircled{4}$ from $\textcircled{3}$

$$19x = -190$$

$$x = -10$$

Substitute $x = -10$ into $\textcircled{1}$

$$5y - 3(-10) = 30$$

$$y = 0$$

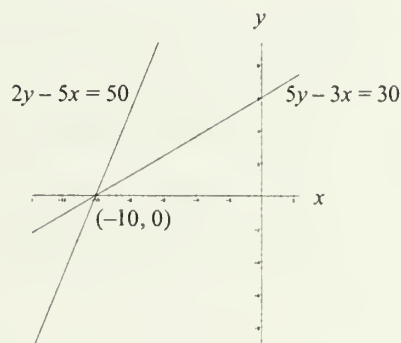
Solution is

$$x = -10$$

$$y = 0.$$

The solution is $(-10, 0)$

Graphical



(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Solve systems of linear equations, in two variables:

- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

3. Solve algebraically:

$$\frac{1}{4}x + \frac{3}{5}y = \frac{2}{3} \quad \textcircled{1}$$

$$\frac{2}{5}x + \frac{1}{5}y = \frac{3}{4} \quad \textcircled{2}$$

Solution:

3. Multiply $\textcircled{1}$ by 60 and $\textcircled{2}$ by 20

$$15x + 36y = 40 \quad \textcircled{3}$$

$$8x + 4y = 15 \quad \textcircled{4}$$

Multiply $\textcircled{4}$ by 9

$$15x + 36y = 40 \quad \textcircled{3}$$

$$72x + 36y = 135 \quad \textcircled{5}$$

Subtract $\textcircled{5}$ from $\textcircled{3}$

$$-57x = -95$$

$$x = \frac{-95}{-57} = \frac{5}{3}$$

Substitute $x = \frac{5}{3}$ into $\textcircled{3}$

$$15\left(\frac{5}{3}\right) + 36y = 40$$

$$25 + 36y = 40$$

$$y = \frac{15}{36} = \frac{5}{12}$$

$$x = \frac{5}{3} \text{ and } y = \frac{5}{12}$$

Question:

4. Three pencils and four erasers cost \$2.50, while one pencil and three erasers cost \$1.25. Find the cost of a pencil and the cost of an eraser.

Solution:

4. Let the cost of a pencil = P .

Let the cost of an eraser = E .

$$3P + 4E = \$2.50 \text{ and } 1P + 3E = \$1.25.$$

$$(1P + 3E = \$1.25) \times 3$$

$$3P + 9E = \$3.75$$

Subtract:

$$3P + 9E = \$3.75$$

$$-(3P + 4E = \$2.50)$$

$$\frac{5E}{5} = \frac{\$1.25}{5}$$

$$E = \$0.25$$

$$P = \$0.50$$

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Solve systems of linear equations, in two variables:

- algebraically (elimination and substitution)
- graphically.

[CN, PS, T, V]

Sample Tasks

- | | |
|-------|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [] | Problem-solving |

(continued)

Solution:

4. Substitute $E = 0.25$ in

$$3P + 4E = \$2.50$$

$$3P + 4(0.25) = \$2.50$$

$$\frac{3P}{3} = \frac{\$1.50}{3}$$

$$P = \$0.50$$

Each pencil costs \$0.50.

Each eraser costs \$0.25.

Question:

5. Jojo wants to set up an Internet connection with Easy Shop Internet. He has two plans to choose from. Plan A costs \$30.00 per month and includes a user fee of \$0.10 per minute. Plan B costs \$10.00 per month and includes a user fee of \$0.15 per minute.

- What would determine which plan is the most economical?
- When would both plans be equally economical to use? Using the given information, find the solution graphically. Draw the image that appears on your graphing calculator screen, and fill in the blanks below.

$Y_1 =$
$Y_2 =$
$X_{\min} =$
$X_{\max} =$
$Y_{\min} =$
$Y_{\max} =$

- Find the solution algebraically by either substitution or elimination.
- Are the results in b and c the same? Explain.

Solution:

- The number of minutes of use should be used to determine which is most economical.
- Let x represent the air time in minutes. Let y_1 represent the monthly cost for plan A and y_2 the monthly cost for plan B.

$Y_1 = 30 + 0.10 X$
$Y_2 = 10 + 0.15 X$
$X_{\min} = 380$
$X_{\max} = 420$
$Y_{\min} = 68$
$Y_{\max} = 72$

The solution is (400, 70). This means at 400 minutes usage, the cost of both plans is the same.

(continued)

General and Specific Outcomes

General Outcome

Represent and analyze situations that involve expressions, equations and inequalities.

Specific Outcome

3.2 Solve systems of linear equations, in two variables:

- algebraically (elimination and substitution)
 - graphically.
- [CN, PS, T, V]

Sample Tasks

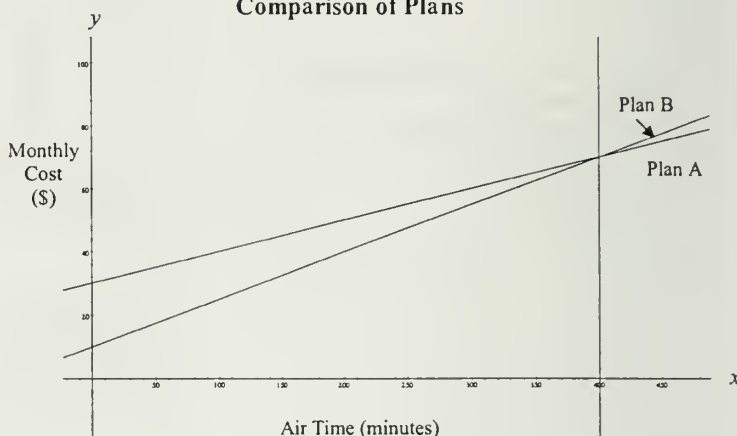
- | | |
|-------|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [] | Problem-solving |

(continued)

Solution:

5.

Comparison of Plans



- c. when the cost is the same
- $$30 + 0.1x = 10 + 0.15x$$
- $$20 = 0.05x$$
- $$400 = x$$
- Each plan costs \$70 for 400 minutes usage.
- d. The solution may not be exactly the same, because the graph is always an approximation.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- solve questions 1 and 2
- graph the equations and find the point of intersection for question 5.

A student demonstrating the *standard of excellence* can also:

- solve questions 3 and 4
- interpret the point of intersection in question 5 as the solution to the problem
- solve the system algebraically in question 5
- explain why the answers for question 5, parts b and c may not be exactly the same.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- Students were introduced to inequalities with one variable in Grade 9 mathematics, where they solved inequalities and graphed inequalities on a number line.
- Coordinate geometry skills are essential for this specific outcome. The concepts of plotting points and intercepts, line graphing, and the use of calculators are the more important concepts to review.
- Many constraint inequalities are of the form $Ax + By = C$, rather than $Ax + By + C = 0$.
- $Ax + By + C = 0$ can be sketched using intercepts.
- Conversion from the $Ax + By + C = 0$ form to any $y =$ form is a necessary preliminary step to the use of a graphing calculator.
- Window settings, on a graphing calculator, are useful to replace the plotting of horizontal and vertical lines. For example, $x < 5$ could be entered into window settings as an x -max of 5.
- Students should first graph manually—to solve for and sketch the solution region—and then graph using the graphing calculator. An even balance between both approaches is recommended.
- It is important for students to first sketch the system of linear inequalities, because the graphing calculator may need adjustments in window settings or in the use of the zoom function to show the intersection point or points of the inequalities.
- With multiple inequalities, students may find it easier to see the intersection by graphing the opposite inequalities, so the intersection region is unshaded.
- Examples should include both open solution regions and closed polygon solution regions.
- Nonlinear inequalities should be expressed as functions.
- Most problem situations have solutions in the first quadrant.
- Price and profit problems can lead to quadratic inequalities.
- Area and perimeter problems often lead to nonlinear inequalities.

General and Specific Outcomes
General Outcome Use linear programming to solve optimization problems. Specific Outcome 3.3 Use expressions containing variables to describe problem contexts and solutions. [C, CN, PS, R]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: - graph the boundary line between two half planes - use a test point, usually (0, 0), to determine the solution region that satisfies the inequality, given a boundary line - graph a linear inequality expressed in the form $y = mx + b$, using $<$, $>$, $\leq$, $\geq$ - rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, where A, B, C are integral and $B > 0$ - explain why the solution region represents the solution to the problem.	A student demonstrating the <i>standard of excellence</i> can also: - distinguish between the use of solid and broken lines in solution regions - graph any linear inequality in two variables - rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, and graph - explain why the shaded half plane represents the solution region of the inequality - explain how the solution region is a combination of different half planes.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

Sample Tasks

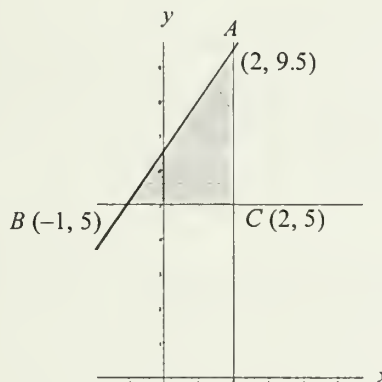
		Conceptual
	✓	Procedural
		Problem-solving

Question:

1. A triangular target in a video game is described in terms of coordinates (x, y) , where x and y are measured in centimetres. The target is defined by the points $A(2, 9.5)$, $B(-1, 5)$ and $C(2, 5)$. What are the equations of the inequalities that define the target?

Solution:

1.



The line BC is horizontal through $(-1, 5)$ and $(2, 5)$, so has equation $y \geq 5$. The line AC is vertical through $(2, 5)$ and $(2, 9.5)$, so has equation $x \leq 2$. The line AB has a slope of $\frac{9.5-5}{2-(-1)} = 1.5$, through the point $(-1, 5)$. Its equation is $y - 5 \leq 1.5(x + 1)$ or $y \leq 1.5x + 6.5$.

(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

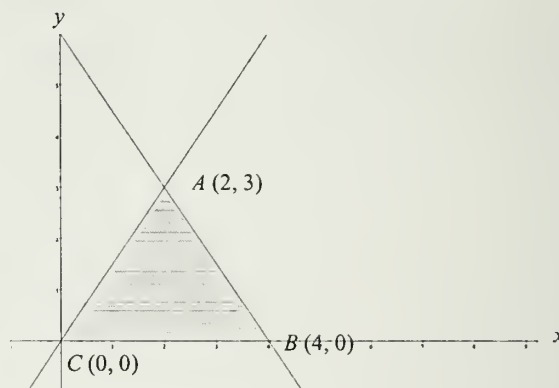
Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [] | Problem-solving |

(continued)

Question:

2. Given the following graph, provide the system of inequalities whose solution is the interior of $\triangle ABC$.



Solution:

2. AC has a slope of $\frac{3}{2}$, through the point $(0, 0)$, so its equation is $y \leq \frac{3}{2}x$.
 AB has slope $= \frac{0-3}{4-2} = -1.5$, and goes through the point $(4, 0)$ so its equation is $y \leq -\frac{3}{2}x + 6$.
 BC is the x -axis, so its equation is $y \geq 0$.

(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

Sample Tasks

		Conceptual
		Procedural
✓		Problem-solving

(continued)

Question:

3. A farmer has chickens and turkeys. He has room for, at most, 100 birds. He sells chickens for \$10 each and turkeys for \$30 each, and plans for revenue income of at least \$1500. Represent the situation graphically, and shade the region containing possible solutions.

Solution:

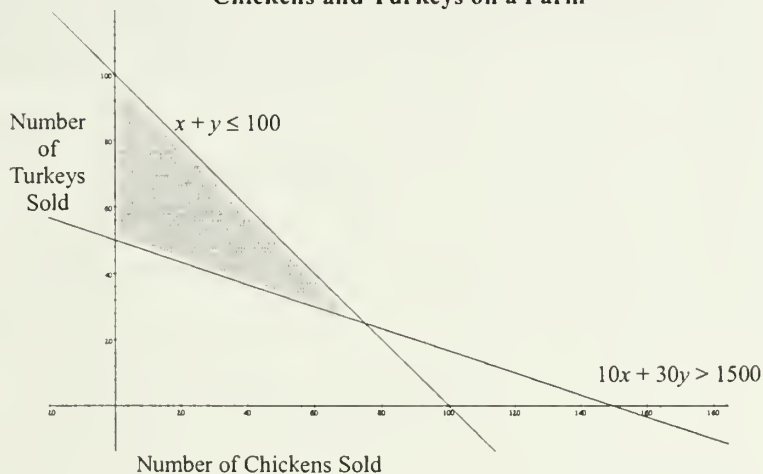
3. Let x be the number of chickens sold.
Let y be the number of turkeys sold.
Conditions for the number of birds:
Rewriting $x + y \leq 100$ in terms of the variable y ,
 $y \leq 100 - x$ $y \geq 0$

Conditions for revenue:

$$10x + 30y > 1500 \quad x \geq 0$$

Rewriting $10x + 30y > 1500$ in terms of the variable y ,
 $y > 50 - \frac{1}{3}x$

Chickens and Turkeys on a Farm



(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

Sample Tasks

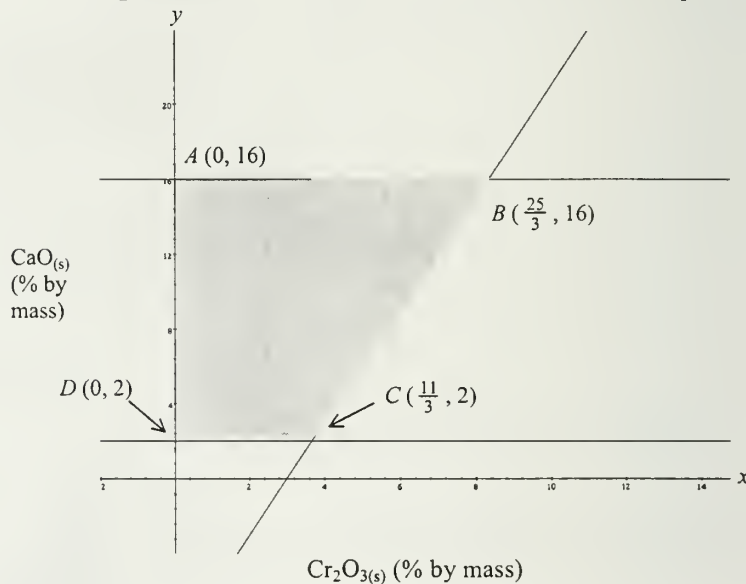
- | | |
|-----|-----------------|
| [] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

(continued)

Question:

4. Diamond prospecting is done by sampling garnets from various places and then testing the garnets for the per cent, by mass, of $\text{CaO}_{(s)}$ and $\text{Cr}_2\text{O}_{3(s)}$. Diamonds occur 85% of the time with garnets classed in the shaded region of the following sketch of a graph.

Using Calcium and Chromium Oxides in Diamond Prospecting



- Determine the system of linear inequalities that determines the shaded area.
- Use the following table and the graph to justify if it is likely that diamonds will be present in the place where each sample was obtained.

Garnet Sample No.	$\text{CaO}_{(s)}$ (% by mass)	$\text{Cr}_2\text{O}_{3(s)}$ (% by mass)
1	10.6	8.4
2	1.1	8.3
3	8.3	1.2
4	15	5.1

(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

Sample Tasks

[]	Conceptual
[]	Procedural
[✓]	Problem-solving

(continued)

Solution:

4. a. AB is horizontal, through $(0, 16)$, so its equation is $y \leq 16$.
 BC has a slope of $\frac{16 - 2}{\left(\frac{25}{3} - \frac{11}{3}\right)} = 3$, through $\left(\frac{11}{3}, 2\right)$, so its equation is $y - 2 \geq 3\left(x - \frac{11}{3}\right)$ or $y \geq 3x - 9$.
 CD is horizontal through $(0, 2)$, so its equation is $y \geq 2$.
 AD is the y -axis, so its equation is $x \geq 0$.
 b. It is not likely that there are diamonds present in the places where samples 1 and 2 were found, because they are not in the shaded region. It is likely that there are diamonds present in the places where samples 3 and 4 were found, because they are in the shaded region.

Question:

5. A desktop publisher has to design formats for rectangular data tables and uses graphing grids as a design tool. Shade the region on the grid that represents the possible dimensions of rectangles in which the length is less than twice the width, the perimeter is at most 48 cm, and the area is at least 32 cm^2 .

Solution:

5. Let y be the length in centimetres.
 Let x be the width in centimetres.
 The length constraint is: $y < 2x$
 The area constraint is: $xy \geq 32; x > 0, y > 0$

$$y \geq \frac{32}{x}$$

 The perimeter constraint is: $2y + 2x \leq 48; y > 0, x > 0$

$$y \leq 24 - x$$

(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.3 Use expressions containing variables to describe problem contexts and solutions.
[C, CN, PS, R]

Sample Tasks

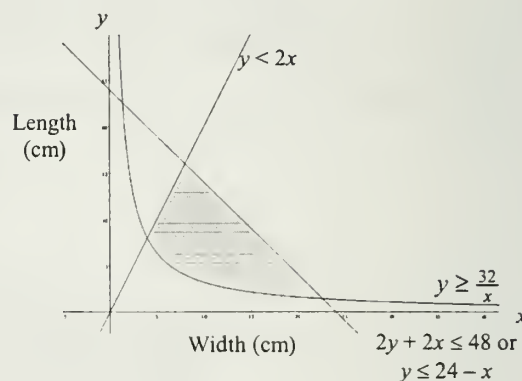
- | | |
|-----|-----------------|
| [] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

(continued)

Solution:

5.

Possible Dimensions of Rectangles



Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- write the equations for two of the boundary lines in question 1
- write the equations for two of the three boundary lines in question 2
- find either inequality in question 3
- find the inequalities describing the vertical and horizontal lines in question 4, part a
- complete question 4, part b
- find the inequality for the perimeter constraint in question 5.

A student demonstrating the *standard of excellence* can also:

- complete question 1
- complete question 2
- solve all of question 3
- complete question 4, part a, using algebra
- solve question 5.

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.4 Solve, graphically, systems of linear inequalities, in two variables, using technology.
[CN, PS, T, V]

Notes:

- Students were introduced to inequalities with one variable in Grade 9 mathematics, where they solved inequalities and graphed inequalities on a number line.
- Coordinate geometry skills are essential for this specific outcome. The concepts of plotting points and intercepts, line graphing, and the use of calculators are the more important concepts to review.
- Many constraint inequalities are of the form $Ax + By = C$, rather than $Ax + By + C = 0$.
- $Ax + By + C = 0$ can be sketched using intercepts.
- Conversion from the $Ax + By + C = 0$ form to any $y =$ form is a necessary preliminary step to the use of a graphing calculator.
- Window settings, on a graphing calculator, are useful to replace the plotting of horizontal and vertical lines. For example, $x < 5$ could be entered into window settings as an x -max of 5.
- Students should first graph manually—to solve for and sketch the solution region—and then graph using the graphing calculator. An even balance between both approaches is recommended.
- It is important for students to first sketch the system of linear inequalities, because the graphing calculator may need adjustments in window settings or in the use of the zoom function to show the intersection point or points of the inequalities.
- With multiple inequalities, students may find it easier to see the intersection by graphing the opposite inequalities, so the intersection region is unshaded.
- Examples should include both open solution regions and closed polygon solution regions.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- graph the boundary line between two half planes
- use a test point to determine the solution region that satisfies the inequality, given a boundary line
- graph a linear inequality expressed in the form $y = mx + b$, using $<$, $>$, $\leq$, $\geq$
- rewrite any inequality expressed in the $Ax + By = C$ form in the $y = mx + b$ form, where A, B, C are integral and $B > 0$
- explain why the solution region represents the solution to the problem.

A student demonstrating the *standard of excellence* can also:

- distinguish between the use of solid and broken lines in solution regions
- graph any linear inequality in two variables
- rewrite any inequality expressed in the $Ax + By = C$ form, where A, B, C can be non-integral with $B \neq 0$, in the $y = mx + b$ form, and graph
- explain why the shaded half plane represents the solution region of the inequality
- explain how the solution region is a combination of different half planes.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

3.4 Solve, graphically, systems of linear inequalities, in two variables, using technology.
[CN, PS, T, V]

Sample Tasks

		Conceptual
		Procedural
		Problem-solving

Question:

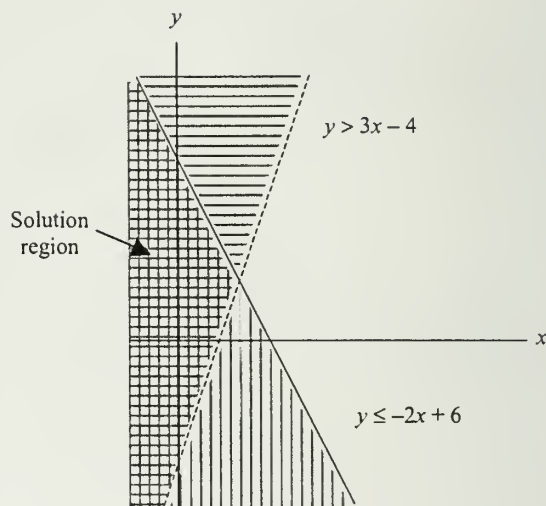
- Graph the solution to the following system of inequalities:

$$y > 3x - 4$$

$$y \leq -2x + 6$$

Solution:

1.



Question:

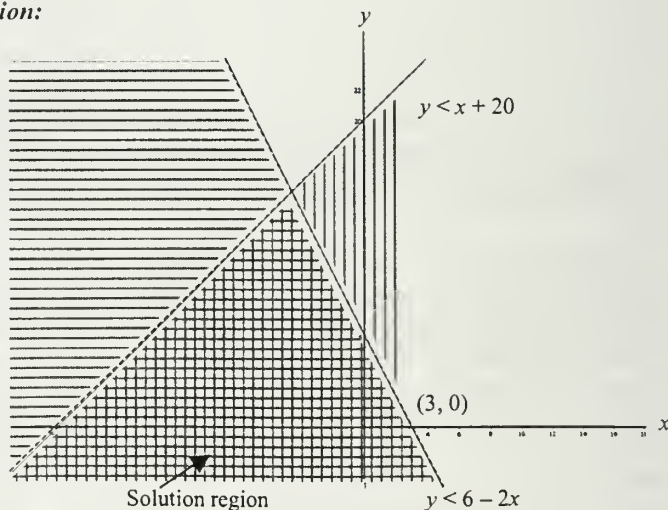
- Graph the solution to the following system of inequalities:

$$y < x + 20$$

$$y < 6 - 2x$$

Solution:

2.



(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.4 Solve, graphically, systems of linear inequalities, in two variables, using technology.
[CN, PS, T, V]

Sample Tasks

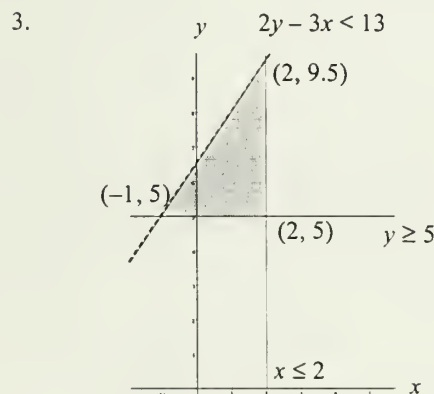
[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

Question:

3. A target in a video game is described in terms of coordinates (x, y) , where x and y are measured in centimetres. All of the following are true:
 $y \geq 5$
 $2y - 3x < 13$
 $x \leq 2$.
 What is the shape and the area of the target?

Solution:



The intersection point of the lines $2y - 3x = 13$ and $y = 5$ is $(-1, 5)$.

The intersection point of the lines $x = 2$ and $y = 5$ is $(2, 5)$.

The intersection point of the lines $x = 2$ and $2y - 3x = 13$ is $(2, 9.5)$.

The target is a right triangle of base 3 cm and height 4.5 cm. Therefore, the area is 6.75 cm^2 .

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- complete question 1
- complete question 2, if guided to sketch first and then use the zoom function
- draw the boundary lines in question 3
- find one of the points in question 3.

A student demonstrating the *standard of excellence* can also:

- complete question 2, without guidance
- complete question 3.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.5 Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and [T] Technology
Mental Mathematics [V] Visualization

Notes:

- Technology [T] is a key mathematical process for this outcome.
- Use window settings for horizontal and vertical lines.
- For problems where the intersection of the inequalities is difficult to distinguish on the graphing calculator, students could reverse the inequality signs and view the intersection as the unshaded region.
- Most problem situations have solutions in the first quadrant.
- Price and profit problems can lead to quadratic inequalities.
- Area and perimeter problems often lead to nonlinear inequalities.
- Intersection points are often found using technology.
- All nonlinear inequalities should be solved using technology.
- Students may find this outcome challenging as a whole; it may be useful to guide students into structured solutions.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- graph the boundary line between two half planes
- use a test point to determine the solution region that satisfies the inequality, given a boundary line.

A student demonstrating the *standard of excellence* can also:

- distinguish between the use of solid and broken lines in solution regions
- graph any linear inequality in two variables
- explain why the shaded half plane represents the solution region of the inequality.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

3.5 Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]

Sample Tasks

		Conceptual
		Procedural
✓		Problem-solving

Question:

- A farmer has chickens and turkeys. He has room for, at most, 100 birds. He sells chickens for \$10 each and turkeys for \$30 each, and plans for revenue income of at least \$1500. Let x be the number of chickens sold. Let y be the number of turkeys sold.
 - Write a system of inequalities to represent the constraints for the number of birds.
 - Write a system of inequalities to represent the conditions for revenue.
 - Graph the systems on the coordinate plane, shading the solution region.
 - State one possible solution.

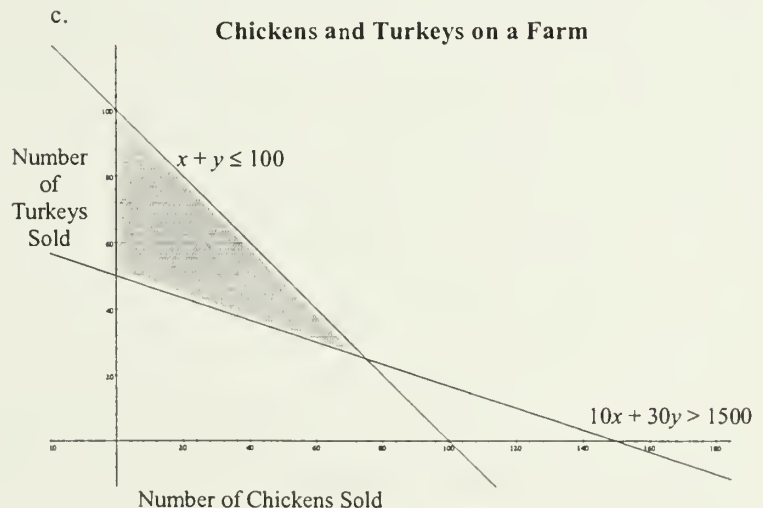
Solution:

- Let x be the number of chickens sold. Let y be the number of turkeys sold. Conditions for the number of birds:
 - Rewriting $x + y \leq 100$ in terms of the variable y ,

$$y \leq 100 - x \quad y \geq 0$$
 - Conditions for revenue:

$$10x + 30y > 1500 \quad x \geq 0$$
Rewriting $10x + 30y > 1500$ in terms of the variable y ,

$$y > 50 - \frac{1}{3}x$$



- 30 chickens and 60 turkeys would be a possible solution.

(continued)

General and Specific Outcomes	
General Outcome	
Use linear programming to solve optimization problems.	
Specific Outcome	
3.5	Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]

Sample Tasks

[]	Conceptual
[]	Procedural
[✓]	Problem-solving

(continued)

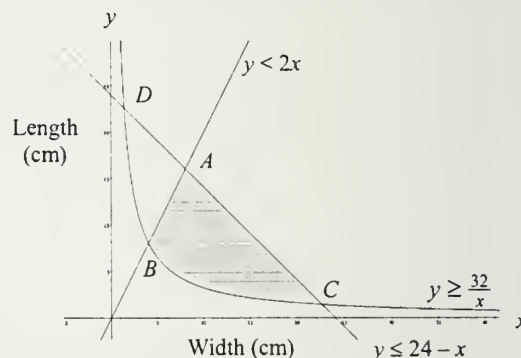
Question:

2. A desktop publisher has to design formats for rectangular data tables and uses graphing grids as a design tool. The length is less than twice the width, the perimeter is at most 48 cm, and the area is at least 32 cm².
 - a. What is represented by the manipulated variable x ?
What is represented by the responding variable y ?
 - b. Write a system of inequalities to represent the length constraint and the area constraint.
 - c. Graph the system and shade the solution region.
 - d. Solve for the three corner points in the solution region.

Solution:

2. a. The variable x could be the width in centimetres.
The variable y could be the length in centimetres.
- b. The length constraint is $y < 2x$.
The perimeter constraint is $2y + 2x \leq 48$.
The area constraint is $xy \geq 32$.

c. **Possible Layout Designs**



(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.5 Design and solve linear and nonlinear systems, in two variables, to model problem situations. [C, CN, PS, R, V]

Sample Tasks

☐	Conceptual
☐	Procedural
☒	Problem-solving

(continued)

Solution:

2. d. Point A is the intersection point of $y = 2x$ and $y = 24 - x$, so $2x = 24 - x$ and $x = 8$.
 If $x = 8$, $y = 2x = 2(8) = 16$.
 $\therefore A$ is $(8, 16)$.
 Point B is the intersection point of $y = 2x$ and $y = \frac{32}{x}$, so $2x = \frac{32}{x}$
 or $x^2 = 16$, and $x = 4$.
 If $x = 4$, $y = 2x = 2(4) = 8$.
 $\therefore B$ is $(4, 8)$.
 Point C is the intersection point of $y = 24 - x$ and $y = \frac{32}{x}$. These must be graphed. The intersection point is $(22.39, 1.61)$, so C is $(22.39, 1.61)$. The solution $(1.61, 22.39)$ is the point D outside the solution region.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- solve question 1
- complete question 2, parts a and b.

A student demonstrating the *standard of excellence* can also:

- solve question 2, parts c and d.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

3.6 Apply linear programming to find optimal solutions to decision-making problems.

[C, PS, R, T, V]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- The optimal solution is found by substituting the coordinates of the vertices into the expression for the objective function.
- The optimal solution must lie within the domain of the relation; e.g., if x represents the number of people, then x must be a whole number.
- In addition to the stated restrictions, other inequalities may be implicit and must be considered to find the solution; e.g., if x represents the length of a rectangle, then $x > 0$.
- The optimal solution may involve either real numbers or natural numbers, depending on the context of the problem.
- Vertices should be part of the domain for all assessment problems.
- Students may find this outcome challenging as a whole; it may be useful to start by introducing structured solutions.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- write the system of inequalities corresponding to a problem context
- graph inequalities
- find vertices
- write an expression for the objective function
- substitute vertices into an expression for the objective function to find the optimal solution.

A student demonstrating the *standard of excellence* can also:

- integrate the elements of the solution process to find the optimal solution to a decision-making problem
- summarize and explain why the solution is correct.

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.6 Apply linear programming to find optimal solutions to decision-making problems.
[C, PS, R, T, V]

Sample Tasks

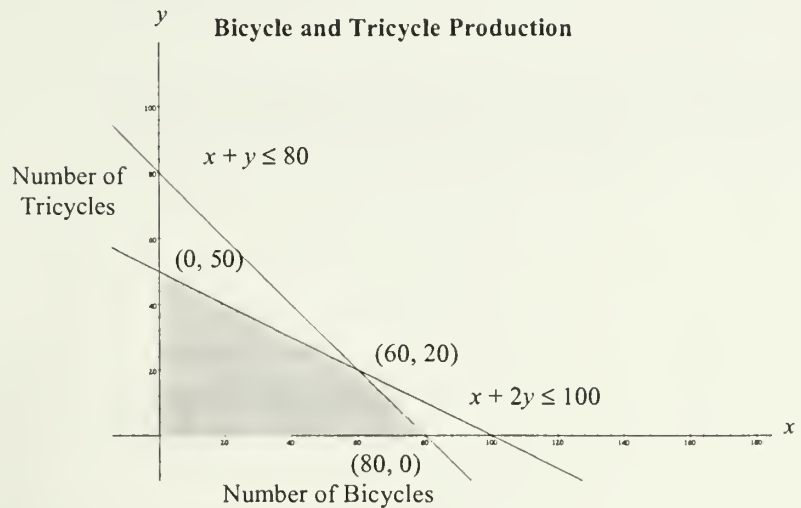
		Conceptual
		Procedural
✓		Problem-solving

Question:

- A firm manufactures bicycles and tricycles, making a profit of \$50 on each bicycle and \$30 on each tricycle. Find the maximum monthly profit that may be made based on the following conditions.
 - The total number of frames made cannot exceed 80 per month.
 - It takes 1 h to assemble a bicycle but 2 h to assemble a tricycle. The assembly machine is available for only 100 h each month.

Solution:

- Let x be the number of bicycles.
Let y be the number of tricycles.
The profit function is $P = 50x + 30y$.
The constraint inequalities are:
 $x + y \leq 80$ for frames, and
 $x + 2y \leq 100$ for assembly.
($x \geq 0$ for domain, $y \geq 0$ for range).



Testing vertices

$$\begin{aligned}
 (0, 50) &: P = 50(0) + 30(50) = \$1500 \\
 (60, 20) &: P = 50(60) + 30(20) = \$3600 \\
 (80, 0) &: P = 50(80) + 30(0) = \$4000
 \end{aligned}$$

Optimal solution: 80 bicycles, no tricycles for \$4000.

(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.6 Apply linear programming to find optimal solutions to decision-making problems.
[C, PS, R, T, V]

Sample Tasks

☐	☐	Conceptual
☐	☐	Procedural
☒	☐	Problem-solving

(continued)

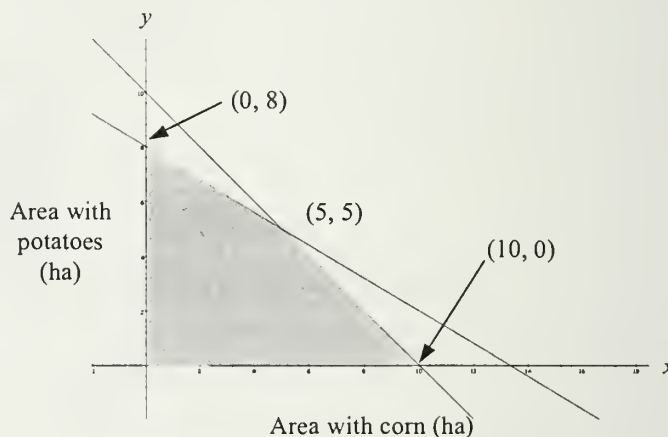
Question:

2. An agricultural club has a 10 ha plot of land available for a market garden project. It has selected corn and potatoes to plant and has \$4000 for the project. The corn will cost \$300/ha to grow and will generate \$375/ha gross income. The potatoes will cost \$500/ha to grow and will generate \$650/ha gross income.
 - a. Construct the function that describes the revenue from the project.
 - b. Construct the inequalities that describe the restrictions.
 - c. Graph this system of inequalities.
 - d. Identify the feasible solutions.
 - e. Determine the optimal solution, and explain why it is optimal.

Solution:

2. Let x be the area of corn in ha.
Let y be the area of potatoes in ha.
 - a. Revenue = $375x + 650y$
 - b. $x + y \leq 10$ or $y \leq 10 - x$
 $300x + 500y \leq 4000$ or $y \leq 8 - \frac{3}{5}x$
 $x \geq 0$
 $y \geq 0$
 - c.

Corn and Potato Areas



(continued)

General and Specific Outcomes

General Outcome

Use linear programming to solve optimization problems.

Specific Outcome

- 3.6 Apply linear programming to find optimal solutions to decision-making problems.
[C, PS, R, T, V]

Sample Tasks

[]	Conceptual
[]	Procedural
[✓]	Problem-solving

(continued)

Solution:

2. d. Students can solve the system of equations, $y = 8 - \frac{3}{5}x$ and $y = 10 - x$, through substitution or elimination to determine the vertex (5, 5) of the region. The vertex (0, 8) can be found through the slope-intercept form of $y = -\frac{3}{5}x + 8$ or through substitution of $x = 0$. The vertex (10, 0) can be found through substitution of $y = 0$ in the equation $y = 10 - x$. The vertices (5, 5), (0, 8) and (10, 0) are the feasible solutions.

- e. Test (0, 8)
 $R = 375(0) + 650(8)$
 $R = 5200$

Test (10, 0)
 $R = 375(10) + 650(0)$
 $R = 3750$

Test (5, 5)
 $R = 375(5) + 650(5)$
 $R = 5125$

The optimal solution would be 0 ha for corn and 8 ha for potatoes. This maximizes the high-profit potatoes and minimizes the low-profit corn. Not all 10 ha has to be planted.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- complete question 1, if given the constraint inequalities
- complete any three of the five parts of question 2.

A student demonstrating the *standard of excellence* can also:

- complete question 1, including finding the constraint inequalities
- complete question 2 and explain why the solution is optimal.

STANDARDS IN FINANCE

GENERAL OUTCOME:

- Solve consumer problems, using arithmetic operations.

SPECIFIC OUTCOMES:

4.1 Solve consumer problems, including:

- wages earned in various situations
- property taxation
- exchange rates
- unit prices.

[CN, E, PS, R, T]

4.2 Reconcile financial statements, which may include:

- cheque books and bank statements
- credit card statements
- loan statements.

[CN, PS, T]

4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

4.4 Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

Strand: Number (Number Operations)

Students will:

- demonstrate an understanding of and proficiency with calculations
- decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.1 Solve consumer problems, including:
- wages earned in various situations
 - property taxation
 - exchange rates
 - unit prices.
- [CN, E, PS, R, T]

[C] Communication [PS] Problem Solving
 [CN] Connections [R] Reasoning
 [E] Estimation and [T] Technology
 Mental Mathematics [V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 6.1.
- This specific outcome lends itself to a project approach.
- The vocabulary associated with taxation should be reviewed.
- There are two approaches to determining Canada Pension Plan (CPP) and Employment Insurance (EI) deductions—one can use either tables or percentage rates. Rates and maximum values change each year.
- The solution to Question 3 uses 2000 rates for CPP and EI. Later rates are higher for CPP and lower for EI.
- Students need to be made aware of the most appropriate technology to use in a given context.
- Currency exchange rates are updated daily in the media.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- Spreadsheets can be used to meet this outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- calculate unit prices
- calculate wages, including minimum wage rates, regular and overtime pay, and combined salary and commission
- calculate wages involving gratuities and piecework
- solve single-step exchange rate problems.

A student demonstrating the *standard of excellence* can also:

- calculate wages using graduated commission structures
- solve multistep exchange rate problems.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.1 Solve consumer problems, including:
- wages earned in various situations
 - property taxation
 - exchange rates
 - unit prices.
- [CN, E, PS, R, T]

Sample Tasks

- [] Conceptual
[] Procedural
[✓] Problem-solving

Question:

- Employees in Sarah's Clothing Boutique wanted an increase in pay. The owner offered them two options. They can either earn \$1.50 more per hour or have their hourly rate of pay increased by 17%. Maria earns \$7.00 an hour, and Cathy earns \$9.75 an hour.
 - Which option should each employee choose in order to maximize the hourly rate of pay? Explain your answer.
 - At what hourly wage would it be beneficial to take the 17% increase rather than the \$1.50/hour increase?

Adapted with permission from Manitoba Education, *Consumer Math*, Senior 2, page I-C-33.

Solution:

Employee	Option A (+ \$1.50/h)	Option B (+17%)
1. a. Maria	\$8.50	\$8.19
Cathy	\$11.25	\$11.41

Maria should choose Option A, and Cathy should choose Option B.

- Using a spreadsheet or a graph, the student can generate or plot the data to determine that at \$8.83/h it is more beneficial to take the 17% increase.

Question:

- A person travelling from Canada to the U.S. received an exchange rate of \$0.65 U.S. for \$1.00 Canadian. When he returned to Canada and exchanged his U.S. money for Canadian dollars, he expected to receive \$1.35 Canadian for each U.S. dollar. Why did this not happen?

Solution:

- He will receive \$1.54 when he returns to Canada because

$$\frac{\$1 \text{ Canadian}}{\$0.65 \text{ U.S.}} = \frac{x(\$ \text{ Canadian})}{\$1 \text{ U.S.}}$$

The expected \$1.35 Canadian assumes that exchange rates are sums and differences, not ratios.

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.1 Solve consumer problems, including:
- wages earned in various situations
 - property taxation
 - exchange rates
 - unit prices.
- [CN, E, PS, R, T]

Sample Tasks

		Conceptual
✓		Procedural
		Problem-solving

(continued)

Question:

3. Rick earns an annual gross salary of \$39 500. He contributes \$5000 annually into an RRSP at work and pays \$400/year for parking at work.
- Calculate his annual taxable earnings.
 - Estimate his annual net earnings. Is this estimate high or low?

Solution:

$$\begin{aligned} 3. \quad a. \quad \text{Taxable income} &= \$39\,500 - \$5000 \\ &= \$34\,500 \end{aligned}$$

b.

$$\begin{aligned} EI &= \$39\,500 \times 0.03 \\ &= \$1185 \end{aligned}$$

Since \$1185 is above the maximum amount, use \$936 for EI.

$$\begin{aligned} CPP &= \$39\,500 \times 0.03 \\ &= \$1185 \end{aligned}$$

$$\begin{aligned} \text{Income Tax} &= \$34\,500 \times 0.29 \\ &= \$10\,005 \end{aligned}$$

$$\begin{aligned} \text{Total deductions} &= \$400 + \$5000 + \$936 + \$1185 + \$10\,005 \\ &= \$17\,526 \end{aligned}$$

$$\begin{aligned} \text{Net annual earnings} &= \$39\,500 - \$17\,526 \\ &= \$21\,974 \end{aligned}$$

Rick's net annual earnings will be about \$22 000. This estimate is likely to be low as the 29% income tax rate is high for taxable incomes near \$30 000.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- complete, correctly, all parts of question 1
- calculate the reciprocal exchange rate in question 2
- calculate CPP, taxable income and income tax in question 3.

A student demonstrating the *standard of excellence* can also:

- explain why in question 2
- calculate the EI deduction and evaluate the estimate in question 3.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.2 Reconcile financial statements, which may include:

- cheque books and bank statements
- credit card statements
- loan statements.

[CN, PS, T]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 6.2.
- Consider examples, such as credit card statements or accounting associated with fund raising for school activities. Examples: graduation ticket sales, yearbook sales, European tours, dances.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- Students can also analyze credit card or loan statements for this specific outcome.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- perform the necessary calculations to reconcile a given financial statement, and explain the processes involved
- explain the entries on a financial statement
- demonstrate an awareness of bank charges; e.g., overdraft protection fee, interest, service charges, automated teller machine (ATM) charges
- calculate credit charges and balances due on credit card statements.

A student demonstrating the *standard of excellence* can also:

- determine which bank plan is the most economical for a given situation.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.2 Reconcile financial statements, which may include:

- cheque books and bank statements
- credit card statements
- loan statements.

[CN, PS, T]

Sample Tasks

☐	Conceptual
☒	Procedural
☐	Problem-solving

Question:

1. a. Reconcile the following cheque book record with the accompanying bank statement.

Cheque book record:

Cheque number	Date	Description of Transaction	Withdrawal	Deposit	Balance
					\$380.57
457	Sept 1	Car Payment	\$187.50		
458	Sept 10	Car insurance—to Dad	\$50.00		
	Sept 12	Gas deposit—from Joe		\$12.00	
	Sept 11	Cash withdrawal	\$40.00		
	Sept 15	Paycheque		\$144.88	
459	Sept 18	Gift for Maxine	\$18.77		
460	Sept 20	Registration fee—school	\$50.00		
461	Sept 20	Phone bill	\$18.45		
	Sept 22	Cash for weekend	\$50.00		
	Sept 30	Paycheque		\$144.88	
462	Sept 30	New tire for truck	\$63.25		

Bank statement:

Date	Transaction Type	Debit	Credit	Balance
	Balance forward			\$380.57
Sept 7	Cheque 457	\$187.50		\$193.07
Sept 11	Cash withdrawal	\$40.00		\$153.07
Sept 12	Deposit		\$12.00	\$165.07
Sept 15	Deposit		\$144.88	\$309.95
Sept 22	Cash withdrawal	\$50.00		\$259.95
Sept 25	Cheque 461	\$18.45		\$241.50
Sept 26	Cheque 459	\$18.77		\$222.73
Sept 30	Deposit		\$144.88	\$367.61
Sept 30	Service charge	\$2.75		\$364.86
Sept 30	Maintenance Charge	\$1.00		\$363.86

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.2 Reconcile financial statements, which may include:

- cheque books and bank statements
- credit card statements
- loan statements.

[CN, PS, T]

Sample Tasks

☐	Conceptual
☒	Procedural
☐	Problem-solving

(continued)

Question:

1. b. The bank provides the following service charge alternatives to the person with the account above:

Plan A: \$8.50 per month flat fee

Plan B: \$1.00 per month maintenance fee

\$0.65 per cheque

\$0.40 per ATM transaction.

Which plan would be the most economical for this situation?

- c. When would this not be the best alternative?

Solution:

1. a. Cheque book record:

Cheque number	Date	Description of Transaction	Withdrawal	Deposit	Balance
					\$380.57
457	Sept 1	Car Payment	187.50		193.07
458	Sept 10	Car insurance—to Dad	50.00		143.07
	Sept 12	Gas deposit—from Joe		12.00	155.07
	Sept 11	Cash withdrawal	40.00		115.07
	Sept 15	Paycheque		144.88	259.95
459	Sept 18	Gift for Maxine	18.77		241.18
460	Sept 20	Registration fee—school	50.00		191.18
461	Sept 20	Phone bill	18.45		172.73
	Sept 22	Cash for weekend	50.00		122.73
	Sept 30	Paycheque		144.88	267.61
462	Sept 30	New tire for truck	63.25		204.36

Reconciliation: Take the statement balance and subtract any outstanding debits and add any outstanding credits to get an adjusted statement balance. Take your cheque register balance and subtract the service charges. This balance should be the same as the adjusted statement balance.

For this example:

Statement: $\$363.86 - \$50.00 - \$50.00 - \$63.25 = \$200.61$

Cheque Register: $\$204.36 - \$2.75 - \$1.00 = \$200.61$

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.2 Reconcile financial statements, which may include:

- cheque books and bank statements
- credit card statements
- loan statements.

[CN, PS, T]

Sample Tasks

		Conceptual
	✓	Procedural
		Problem-solving

(continued)

Solution:

1. b. Plan A: \$8.50
 Plan B: 6 cheques @ \$0.65 \$3.90
 2 ATM withdrawals @ \$0.40 0.80
 3 ATM deposits @ \$0.40 1.20
 Maintenance fee 1.00
 \$6.90

This assumes that there is a charge for depositing money through an ATM. Most banks do not charge for this, and an answer of \$5.70 is acceptable. In any case, Plan B would be the most economical.

- c. Plan A would become more economical if you regularly wrote more than 11 cheques, made more than 18 ATM transactions, or did a combination of these things that would exceed \$7.50.

Question:

2. Complete the table below to determine the cost of credit for using a department store charge account for the period shown. Monthly credit charges are 1.4% of the balance due.

Month	Previous Balance	- Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance
February	\$314.65	\$100.00	\$193.75		\$5.72	\$414.12
March		\$150.00	\$ 59.60			
April		\$140.00	\$421.83			\$618.62
May	\$618.62	\$200.00	\$ 39.65			
June		\$250.00	\$ 58.11			
July		\$150.00	\$ 77.21			

Solution:

2.

Month	Previous Balance	- Payment Made	+ Purchases Charged	⇒ Balance Due	+ Credit Charges	⇒ New Balance
February	\$314.65	\$100.00	\$193.75	\$408.40	\$5.72	\$414.12
March	\$414.12	\$150.00	\$ 59.60	\$323.72	\$4.53	\$328.25
April	\$328.25	\$140.00	\$421.83	\$610.08	\$8.54	\$618.62
May	\$618.62	\$200.00	\$ 39.65	\$458.27	\$6.41	\$464.68
June	\$464.68	\$250.00	\$ 58.11	\$272.79	\$3.81	\$276.60
July	\$276.60	\$150.00	\$ 77.21	\$203.81	\$2.84	\$206.65

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- complete question 1, parts a and b
- complete question 2.

A student demonstrating the *standard of excellence* can also:

- justify when it would be more economical to go to Plan A in question 1, part c.

General and Specific Outcomes
General Outcome Solve consumer problems, using arithmetic operations.
Specific Outcome 4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 6.3.
- Consider budget situations that involve only a few variables. This specific outcome could easily expand into a course itself.
- This specific outcome lends itself easily to a project approach; applied resources have many examples of projects that reflect this specific outcome.
- Specific outcomes in Career and Life Management 20 have some commonality with this specific outcome.
- Computer budget programs would be excellent tools for developing budgets.
- This specific outcome lends itself to the use of spreadsheets but can be accomplished using the table function of the graphing calculator or using paper and pencil.
- The graphing capabilities within many spreadsheet programs are recommended for the graphical representation of budget problems and solutions.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: • construct a budget • draw and interpret graphs for the purpose of displaying budget data—limited to circle graphs, bar graphs, pictographs • make predictions based on given data • adapt a budget to reflect a single change in situation.	A student demonstrating the <i>standard of excellence</i> can also: • adapt a budget to multiple changes in situation.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

Question:

1. Yasha completed a class project on the expenses associated with moving away from home and into her own apartment with one room-mate. The following is a summary of her findings:

Item	Total Cost
Rent	\$600.00
Utilities	\$48.00
Food	\$175.00/person
Cable TV	\$25.00
Renter's insurance	\$35.00
Telephone	\$22.50
Transportation	\$110.00/person
Clothing/personal	\$120.00/person

- a. Create a spreadsheet that will calculate Yasha's costs. Use the graphing capability of your spreadsheet to draw a circle graph that depicts Yasha's costs.
- b. If Yasha had another room-mate move in, what expenses would change for her? Adjust your spreadsheet to create a second circle graph to depict the new costs.

Solution:

1. a.

Item	Total Cost	Yasha's Cost
Rent	\$600.00	\$300.00
Utilities	\$48.00	\$24.00
Food	\$350.00	\$175.00
Cable TV	\$25.00	\$12.50
Renter's insurance	\$35.00	\$17.50
Telephone	\$22.50	\$11.25
Transportation	\$220.00	\$110.00
Clothing/personal	\$240.00	\$120.00
		\$770.25

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

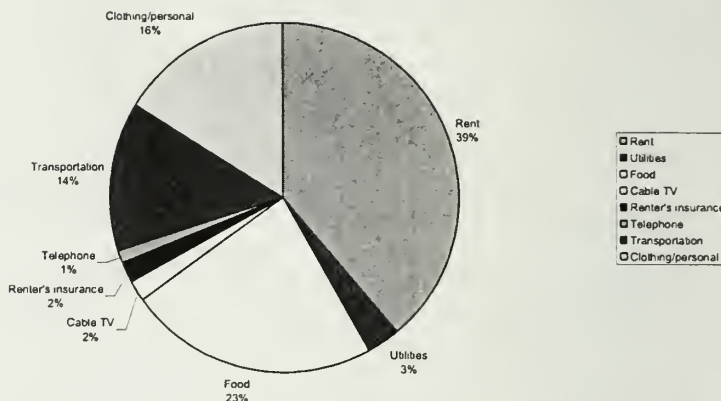
		Conceptual
✓		Procedural
✓		Problem-solving

(continued)

Solution:

1. a.

Yasha's Costs



b. The new table is as follows.

Item	Total Cost	Yasha's Cost
Rent	\$600.00	\$200.00
Utilities	\$48.00	\$16.00
Food	\$525.00	\$175.00
Cable TV	\$25.00	\$8.33
Renter's insurance	\$35.00	\$11.67
Telephone	\$22.50	\$7.50
Transportation	\$330.00	\$110.00
Clothing/personal	\$360.00	\$120.00
		\$648.50

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

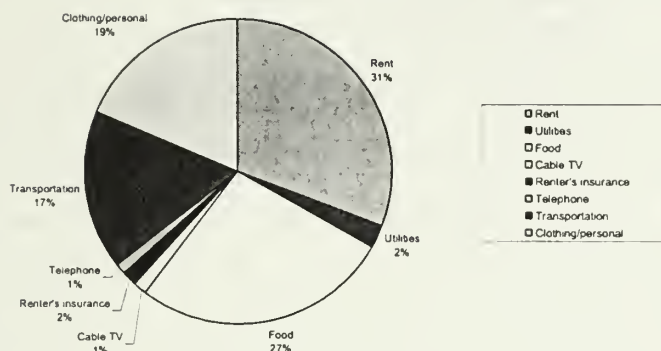
[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:

1. b.

Expenses with Two Room-mates



Question:

- Imagine yourself as a student who will graduate from a post-secondary program next year. You expect to be offered a permanent job and have decided you will buy a car and live independently in an apartment. As a student, you have been sharing an apartment with two room-mates and have been using public transportation. You will also have a student loan to pay back when you graduate. Develop two budgets: one to reflect your expenses as a student, another to reflect your expenses as a worker with a full-time job. Explain the choices you make in each budget.

Solution:

- Answers will vary, but should reflect current costs and salaries and may or may not include deductions from salaries. A sample answer might be:

Expense Category	Expenses per month
Rent (one-third share)	\$175
Food (at home and out)	\$180
Clothing/personal	\$110
Transportation (bus)	\$90
Renter insurance	\$25
Student loan expenses	\$5
Tuition and books	\$350
Deductions from part-time job	\$60
Total	\$995

(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

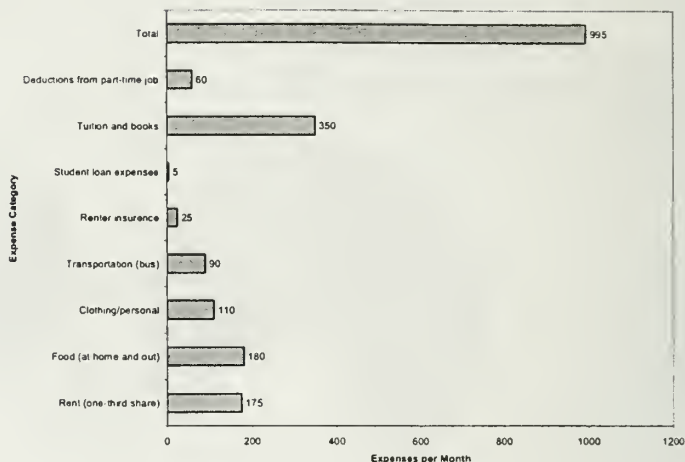
[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:

2.

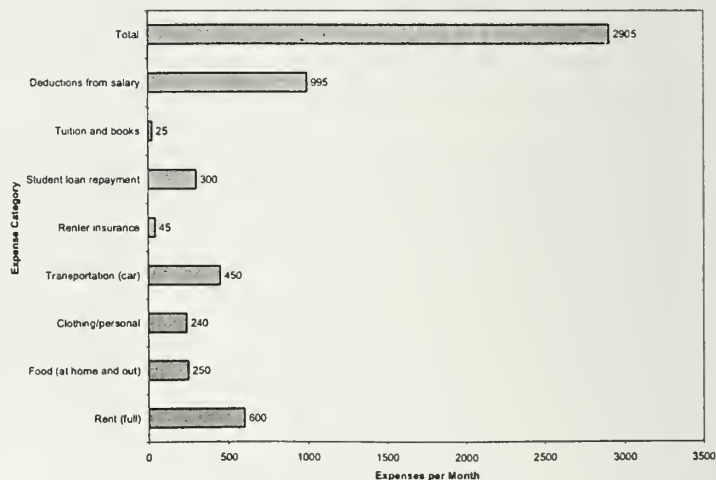
Student Budget



Worker with full-time job:

Expense Category	Expenses per month
Rent (full)	\$600
Food (at home and out)	\$250
Clothing/personal	\$240
Transportation (car)	\$450
Renter insurance	\$45
Student loan repayment	\$300
Tuition and books	\$25
Deductions from salary	\$995
Total	\$2905

Working Budget



(continued)

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.3 Solve budget problems, using graphs and tables to communicate solutions. [C, PS, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:

2.

Explanations:

- car expenses are based on a used car costing under \$10 000, as the average starting salary will not be enough to afford a new car
- student loan repayment is based on repaying \$20 000 over 7 years
- clothing allowance is higher for a working person compared to a student
- food allowance is higher, as there may be a need to eat out more often as a working person
- no money is set aside for savings or investments

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- translate the numbers in the table in question 1 into percentages or degrees, and show them graphically in question 1, part a
- identify what items would change in question 1, part b; make the necessary changes in the data; and complete the corresponding calculations and graph in question 1, part b
- produce a student budget in question 2.

A student demonstrating the *standard of excellence* can also:

- produce a working budget in question 2, and explain the choices made.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.4 Solve investment and credit problems involving simple and compound interest.
[CN, PS, T]

[C]	Communication	[PS]	Problem Solving
[CN]	Connections	[R]	Reasoning
[E]	Estimation and Mental Mathematics	[T]	Technology
		[V]	Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 6.5.
- **Caution: This specific outcome could be a time trap.**
- The focus in this specific outcome should be on students using technology—spreadsheets, graphing calculator software and computer financial software—to generate solutions to problems.
- Spreadsheet analysis is covered in Applied Mathematics 30.
- Formula usage is limited to compound interest and simple interest.
- Questions need to be carefully selected, based on the technology available to students. Many exercises in the basic resource; e.g., tutorial 4.6, exercise 6, will be difficult for students who do not have access to graphing calculator software.
- Annuities should be limited to the repayment of loans and the final amount of a planned savings scheme. The basic resource goes further than the standard set for Applied Mathematics 20.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- calculate, using a formula, a table of values or a spreadsheet; and distinguish between simple and compound interest when solving problems
- calculate the appropriate value of the interest rate in the compound interest formula, using technology
- calculate an appropriate number of compounding periods in the compound interest formula, using technology.

A student demonstrating the *standard of excellence* can also:

- complete a multistep problem involving investment or credit.

General and Specific Outcomes

General Outcome

Solve consumer problems, using arithmetic operations.

Specific Outcome

- 4.4 Solve investment and credit problems involving simple and compound interest.
[CN, PS, T]

Sample Tasks

		Conceptual
✓		Procedural
✓		Problem-solving

Question:

- Mr. Chow invests \$10 000 in a GIC that provides annual compound interest. In 7 years it is worth \$18 000. What was the interest rate?

Solution:

- Solution 1, using the Time Value of Money (TVM) Solver:

$N = 7$
 $I = 8.759574725$
 $PV = -10\ 000$
 $PMT = 0$
 $FV = 18\ 000$
 $P/Y = 1$
 $C/Y = 1$
 Payment End
 Interest rate = 8.76%

Solution 2, using the compound interest formula:

$$\begin{aligned}
 18\ 000 &= 10\ 000(1+i)^n \\
 1.8 &= (1+i)^7 \\
 1.8^{\frac{1}{7}} &= 1+i \\
 i &= 0.0876 \\
 i \times 100\% &= 8.76\%
 \end{aligned}$$

Solution 3, using the graphing calculator:

$$\begin{aligned}
 18\ 000 &= 10\ 000(1+i)^7 \\
 y_1 &= 18000 \\
 y_2 &= 10000(1+x)^7 \text{ and find the } x\text{-coordinate of the} \\
 &\text{intersection point.} \\
 x &= 0.0876 \\
 i \times 100\% &= 8.76\% \text{ interest rate.}
 \end{aligned}$$

Question:

- A bank offers an interest rate of 8% per year, compounded annually. A second bank offers an interest rate of 8% per year, compounded quarterly. If \$2000 were deposited, for ten years, in each bank, how much more income would be gained in the second bank than in the first?

(continued)

General and Specific Outcomes	
General Outcome	
Solve consumer problems, using arithmetic operations.	
Specific Outcome	
4.4	Solve investment and credit problems involving simple and compound interest. [CN, PS, T]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

Solution:

2. Solution 1, using the TVM Solver

First Bank		Second Bank
N	10	40
I	8	8
PV	-2000	-2000
PMT	0	0
FV	4317.849995	4416.079327
P/Y	1	4
C/Y	1	4
Payment	End	End

The future value at the first bank is \$4317.85; at the second bank, it is \$4416.08.

Difference between the banks:

$$\$4416.08 - \$4317.85 = \$98.23.$$

The amount gained at the second bank would be \$98.23.

Solution 2, using the compound interest formula:

$$\begin{aligned} \text{First bank: } i &= 0.08, n = 10 \\ \therefore A &= 2000(1.08)^{10} \\ &= \$4317.85 \end{aligned}$$

$$\begin{aligned} \text{Second bank: } i &= 0.02, n = 40 \\ \therefore A &= 2000(1.02)^{40} \\ &= \$4416.08 \end{aligned}$$

The second bank pays \$(4416.08 - 4317.85) or \$98.23 more in interest.

Solution 3, using the calculator:

$$\begin{aligned} y_2 - y_1 &= 2000[(1.02)^{40} - (1.08)^{10}] \\ &= \$98.23 \end{aligned}$$

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: • solve question 1 • calculate the \$4317.85 in the first bank for question 2.	A student demonstrating the <i>standard of excellence</i> can also: • complete the solution for question 2.

STANDARDS IN CIRCLE GEOMETRY AND DESIGN

GENERAL OUTCOME:

- Develop and apply the geometric properties of circles and polygons to solve problems.

SPECIFIC OUTCOMES:

- 5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the sum of the interior angles of an n -sided polygon is $(2n - 4)$ right angles.
- [PS, R, T, V]
- 5.2 Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

Strand: Shape and Space (3-D Objects and 2-D Shapes)

Students will:

- describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

- 5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the sum of the interior angles of an n -sided polygon is $(2n - 4)$ right angles.

[PS, R, T, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- This specific outcome is common with Pure Mathematics 20 specific outcome 5.1.
- Specific outcome 5.1 follows from geometry concepts students have studied in junior high mathematics: Grade 7—properties of parallel lines, transformations; Grade 8—classifying polygons; Grade 9—transformation properties, locus definitions, similar and congruent triangles.
- This specific outcome has been made shorter in Applied Mathematics 20, by deleting the component relating to the angle between a tangent and a chord.
- The use of geometry software with animation, such as *Geometer's Sketchpad* or *Cabri*, can save a lot of time.
- If technology is not available, the properties may be confirmed using manual construction methods, but this is very time consuming.
- Computerized drafting programs are not useful in this specific outcome.
- Students should use technology to confirm and explore properties.
- Students should be able to apply the properties without using technology.
- Projects can be used to integrate specific outcomes 5.1 and 5.2.
- Students need to be familiar with the Pythagorean theorem and need to be able to use it to solve for any unknown side.
- Students need to be able to use any trigonometric ratio to solve for an unknown side or angle.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- apply a single circle geometry property to solve for unknown measures
- follow written instructions to conduct an investigation.

A student demonstrating the *standard of excellence* can also:

- apply more than one circle geometry property to solve for unknown measures
- modify or create instructions to conduct a related investigation.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

- 5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the sum of the interior angles of an n -sided polygon is $(2n - 4)$ right angles.

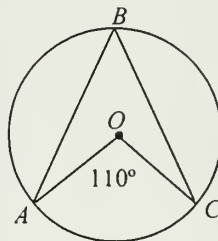
[PS, R, T, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

Question:

- Determine the size of the inscribed $\angle ABC$ on the following diagram. Explain how you arrived at your answer.

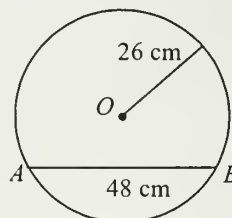


Solution:

- Since the inscribed $\angle ABC$ and the central $\angle AOC$ are subtended by the same arc, then $\angle ABC = \frac{\angle AOC}{2} = 55^\circ$.

Question:

- Find the perpendicular distance from O to AB , given that chord AB is 48 cm. Explain how you arrived at your answer.



Solution:

- Since OB is a radius, it has a length of 26 cm. A line from the centre of the circle, perpendicular to AB , bisects AB . Using the Pythagorean relation, the perpendicular distance from O to AB is 10 cm.

$$26^2 = x^2 + 24^2$$

$$x^2 = 26^2 - 24^2$$

$$x^2 = 100$$

$$x = 10$$
 The perpendicular distance from O to AB is 10 cm.

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.1 Use technology and measurement to confirm and apply the following properties to particular cases:

- the perpendicular from the centre of a circle to a chord bisects the chord
- the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
- the inscribed angles subtended by the same arc are congruent
- the angle inscribed in a semicircle is a right angle
- the opposite angles of a cyclic quadrilateral are supplementary
- a tangent to a circle is perpendicular to the radius at the point of tangency
- the tangent segments to a circle, from any external point, are congruent
- the sum of the interior angles of an n -sided polygon is $(2n - 4)$ right angles.

[PS, R, T, V]

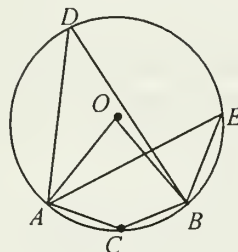
Sample Tasks

		Conceptual
✓		Procedural
		Problem-solving

(continued)

Question:

3. Given A, B, C, D and E are points of a circle with centre O , $\angle AOB = 96^\circ$, find $\angle AEB$ and $\angle ACB$. Explain how you arrived at your answers.



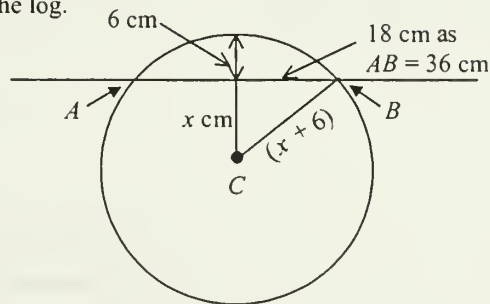
Solution:

3. Since the central $\angle AOB$ and the inscribed $\angle AEB$ are subtended by the same arc, $\angle AEB = \frac{\angle AOB}{2} = 48^\circ$.

$\angle ADB = \angle AEB = 48^\circ$ because they are inscribed angles subtended by the same arc. In the cyclic quadrilateral $ADBC$, $\angle ADB$ is opposite $\angle ACB$ and, therefore, $\angle ADB$ and $\angle ACB$ are supplementary. Therefore, $\angle ACB = 132^\circ$.

Question:

4. A log lies submerged in water such that part of its circular cross-section is sticking out of the water. Find x . Find the diameter of the log.



Solution:

4. $x^2 + 18^2 = (x + 6)^2$
 Solve graphically:
 $x^2 = (x + 6)^2 - 18^2$
 Solve as a system:
 $y_1 = x^2$
 $y_2 = (x + 6)^2 - 18^2$
 $(24, 900)$ is the point of intersection.
 $x = 24$ cm
 $\therefore$ Radius of log = $24 + 6 = 30$ cm.
 Diameter = 60 cm.

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

- 5.1 Use technology and measurement to confirm and apply the following properties to particular cases:
- the perpendicular from the centre of a circle to a chord bisects the chord
 - the measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc
 - the inscribed angles subtended by the same arc are congruent
 - the angle inscribed in a semicircle is a right angle
 - the opposite angles of a cyclic quadrilateral are supplementary
 - a tangent to a circle is perpendicular to the radius at the point of tangency
 - the tangent segments to a circle, from any external point, are congruent
 - the sum of the interior angles of an n -sided polygon is $(2n - 4)$ right angles.

[PS, R, T, V]

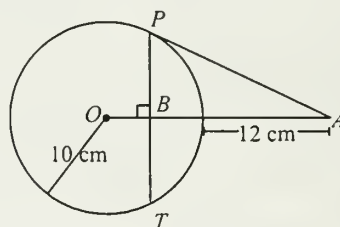
Sample Tasks

		Conceptual
	✓	Procedural
		Problem-solving

(continued)

Question:

5. Given $OB = 4$ cm, find the length of chord PT and AP , when P is a point of tangency, correct to the nearest millimetre. Explain how you arrived at your answers.



Solution:

5. OBA is the perpendicular bisector of the chord PT . The Pythagorean relation can then be used to find PB in $\triangle POB$. PB is 9.165 cm and PT is 18.3 cm. OP is a radius of the circle, so it has a length of 10 cm. The total length of OA is 22 cm. Since AP is a tangent of the circle to the radius OP , $\angle OPA$ is 90° and the Pythagorean relation can be used to find AP . AP is 19.6 cm.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the acceptable standard can:

- provide numerical solutions to questions 1 and 2, with some reference to the diagrams
- calculate $\angle AEB$ in question 3
- solve question 4
- calculate PB or PT in question 5.

A student demonstrating the standard of excellence can also:

- provide complete solutions to questions 1 and 2, with appropriate references to the mathematical circle properties used
- provide complete solutions to questions 3 and 5, with appropriate references to the diagrams.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

[C] Communication [PS] Problem Solving
[CN] Connections [R] Reasoning
[E] Estimation and [T] Technology
Mental Mathematics [V] Visualization

Notes:

- This specific outcome should be strongly linked to specific outcome 5.1.
- This specific outcome could be approached using one or more mini-projects, with students doing one or two for themselves and analyzing the work of others.
- Both manual and technological methods should be encouraged.
- Projects can be done on an individual or group basis.
- Assessment should primarily be based on the development of the mathematical concepts in the design rather than the final aesthetics of the design.
- An individual design project will usually require the application of circle and polygon properties included in grades 7–9, in addition to the properties presented in the previous specific outcome.
- Students should have experience in creating their own designs, analyzing the mathematical content of their own designs and the designs of others, and communicating design concepts to an audience.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- create a simple design, using up to three properties of circles
- communicate, approximately, the properties used in own design
- solve problems that involve properties of polygons or circles
- recognize a property used in another student's design.

A student demonstrating the *standard of excellence* can also:

- create a design, using three or more properties
- identify, precisely, the properties used in own design
- modify an existing design to accommodate additional requirements
- analyze the mathematical content of another student's design.

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

Sample Tasks

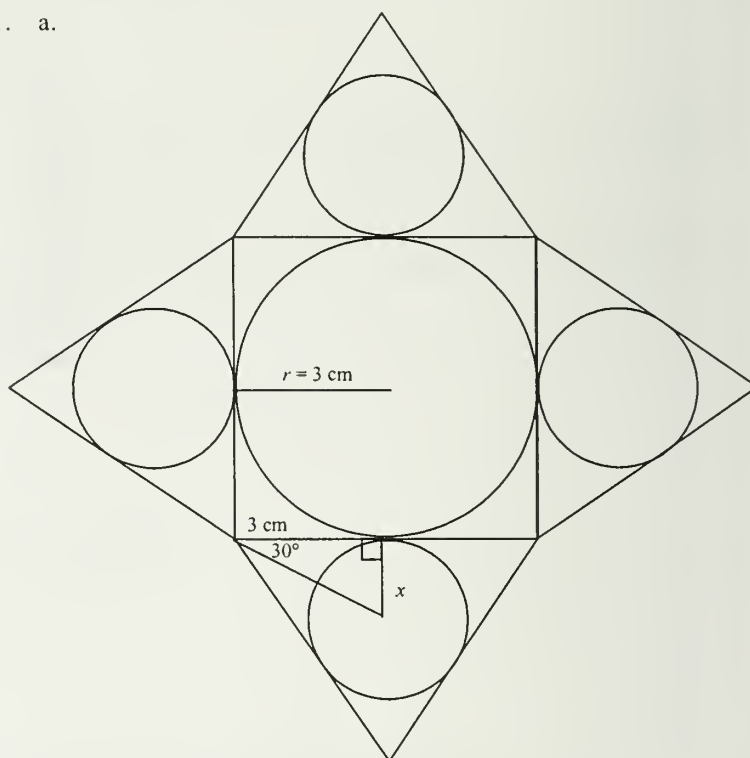
[]	Conceptual
[]	Procedural
[✓]	Problem-solving

Question:

- The pattern on a piece of vinyl flooring consists of a square and four equilateral triangles. Each equilateral triangle has as its base one side of the square. Circles are inscribed in each triangle and in the square.
 - Start with a square that has a side length of 6 cm. Draw the design, full size.
 - Calculate the ratio of the area of the small circle to the area of the large circle.

Solution:

- a.



$$\begin{aligned}
 b. \quad A_1 &= \pi r^2 \\
 &= \pi (3)^2 \\
 &= 28.27 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 \tan 30^\circ &= \frac{x}{3} \\
 x &= 1.732 \text{ cm}
 \end{aligned}$$

$$\begin{aligned}
 A_2 &= \pi (1.732)^2 \\
 &= 9.42 \text{ cm}^2
 \end{aligned}$$

The area of the small circle is $\frac{1}{3}$ the area of the large circle.

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

Sample Tasks

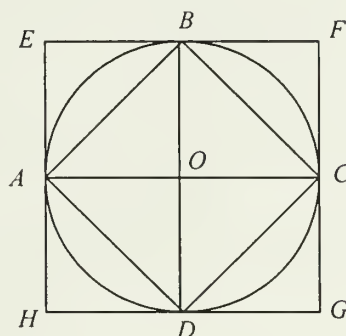
[✓]		Conceptual
[]		Procedural
[✓]		Problem-solving

(continued)

Question:

- Create a scale drawing of a stepping stone that contains a circle with a radius of 15 cm. The design must incorporate three or more of the circle properties. Include diagrams and a full explanation showing how circle properties were used.

Solution: (one of many)



Explanation:

- The measure of the central angle is equal to twice the measure of the inscribed angle subtended by the same arc;
e.g., $\angle DOC$ is twice the measure of $\angle DBC$.
- The inscribed angles subtended by the same arc are congruent;
e.g., $\angle DBC \cong \angle DAC$.
- The angle inscribed in a semicircle is a right angle;
e.g., $\angle ABC$ is a right angle.
- The opposite angles of a cyclic quadrilateral are supplementary;
e.g., $\angle ABC + \angle ADC = 180^\circ$.
- A tangent to a circle is perpendicular to the radius at the point of tangency; e.g., The tangent $\overline{EB}$ is perpendicular to the radius $\overline{OB}$.

(continued)

General and Specific Outcomes

General Outcome

Develop and apply the geometric properties of circles and polygons to solve problems.

Specific Outcome

5.2 Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

Sample Tasks

☐	Conceptual
☐	Procedural
☒	Problem-solving

(continued)

Question:

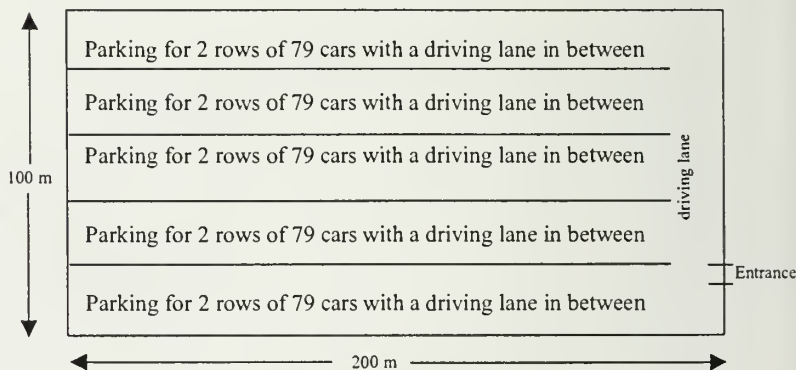
3. A rectangular parking lot at a theme park measures 200 m by 100 m. Parking spots for cars are 7.0 m by 2.5 m. Driving lanes for cars are 5.5 m wide.
 - a. Draw a plan for the parking lot, and determine the number of car parking spots available.
 - b. Modify the design to provide parking for 20 buses, each needing a parking spot that is 18 m long and 4 m wide. Try to minimize the loss of car parking spots.

Solution:

3. a. One possible solution.

Parking Lot for Cars

scale: 1 cm = 20 m



$$\text{Total number of parking spots} = 79 \times 2 \times 5 = 790$$

(continued)

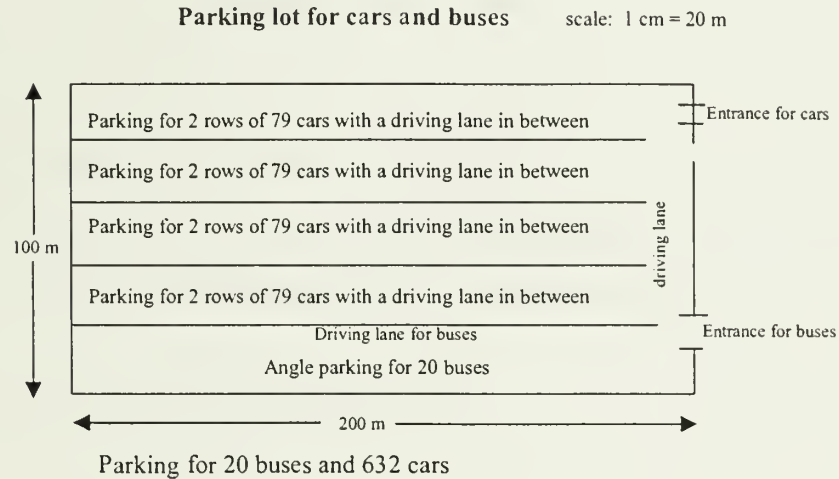
General and Specific Outcomes	
General Outcome	
Develop and apply the geometric properties of circles and polygons to solve problems.	
Specific Outcome	
5.2	Use properties of circles and polygons to solve design and layout problems. [CN, PS, V]

Sample Tasks

[		Conceptual
[		Procedural
[✓		Problem-solving

(continued)
Solution:

3. b. One possible solution.



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: - draw the design in question 1 - choose no more than three circle properties for question 2, and incorporate them into a design that gives minimal attention to aesthetics - provide a sketch of the design for question 2 - develop a workable design for question 3 that includes both driving lanes and parking stalls of reasonable size, and a minimum number of unusable parking spots - calculate or count the number of car parking spots.	A student demonstrating the <i>standard of excellence</i> can also: - calculate the area ratio in question 1 - create a complex, aesthetically pleasing design for question 2, incorporating more than three circle properties - provide an accurate scale drawing of the design for question 2 - justify and explain the particular properties chosen for question 2 - appropriately modify the design in question 3 for four buses.

STANDARDS IN MEASUREMENT AND DESIGN

GENERAL OUTCOMES:

- Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.
- Use measuring devices to make estimates and to perform calculations in solving problems.

SPECIFIC OUTCOMES:

- 6.1 Enlarge or reduce a dimensioned object, according to a specified scale. [C, CN, PS, V]
- 6.2 Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. [PS, R, V]
- 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors, using either formulas or first principles. [PS, R, V]
- 6.4 Design an appropriate measuring process or device to solve a problem. [E, PS, V]

Strand: Shape and Space (Measurement)

Students will:

- describe and compare everyday phenomena, using either direct or indirect measurement.

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

- 6.1 Enlarge or reduce a dimensioned object, according to a specified scale.
[C, CN, PS, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes:

- This specific outcome builds on outcomes in Applied Mathematics 10.
- Students should be comfortable with measuring tools.
- Students have to be able to read and interpret scales.
- 3-D manipulatives will be very helpful in this section.
- Proportion and ratio competency is a requirement.
- Conversion skills within a system of measurement are required.
- The processes of estimation and visualization are emphasized in this outcome.
- For information on significant digits, refer to the Alberta Learning web site at <<http://ednet.edc.gov.ab.ca/studenteval/>>. Look under Diploma Exams, then Essential Information.
- Vocabulary distinctions are less important than the concepts themselves; e.g., the distinction between scale and scale factor.
- Isometric dot or line paper would assist students in drawing 3-D objects.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- read and interpret scale drawings
- enlarge or reduce 1-D or 2-D drawings
- enlarge or reduce simple shapes in 3-D, using orthographic drawings.

A student demonstrating the *standard of excellence* can also:

- enlarge or reduce complex/hidden shapes in 3-D, using orthographic drawings.

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

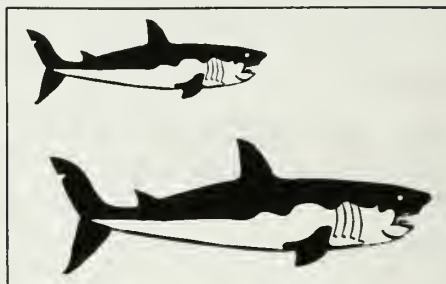
Specific Outcome

- 6.1 Enlarge or reduce a dimensioned object, according to a specified scale.
[C, CN, PS, V]

Sample Tasks

- | | |
|---|-----------------|
| ✓ | Conceptual |
| ✓ | Procedural |
| ✓ | Problem-solving |

Question:



Scale: 1 cm = 0.5 m

1. a. Convert the scale to a ratio, using the same unit of measure.
- b. What are the actual lengths of the two sharks—from the tip of the nose to the middle of the tail—to the nearest tenth metre?
- c. If the above picture was to be drawn on a poster that is 2 m long and 1 m wide, justify the scale factor you would use to adequately fit this space.

Solution:

1. a. Scale is 1 cm = 0.5 m
Convert 0.5 m to 50 cm
Ratio = $\frac{1 \text{ cm}}{50 \text{ cm}}$ or 1:50.

- b. Big Shark:
 $\frac{1}{50} = \frac{5.3 \text{ cm}}{x}$
 $x = 265 \text{ cm}$
 $x = 2.7 \text{ m}$

Small Shark:
 $\frac{1}{50} = \frac{3.4 \text{ cm}}{x}$
 $x = 170 \text{ cm}$
 $x = 1.7 \text{ m}$

- c. Answers may vary, but a scale of 1 m = 2 m would fill the poster page adequately; this is based on the big shark being 1.35 m long, allowing about 30 cm on each end. The more complex scale of 1 m = 1.5 m will fill the poster more completely.

(continued)

General and Specific Outcomes

General Outcome

Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects.

Specific Outcome

- 6.1 Enlarge or reduce a dimensioned object, according to a specified scale.
[C, CN, PS, V]

Sample Tasks

[]	Conceptual
[✓]	Procedural
[]	Problem-solving

(continued)

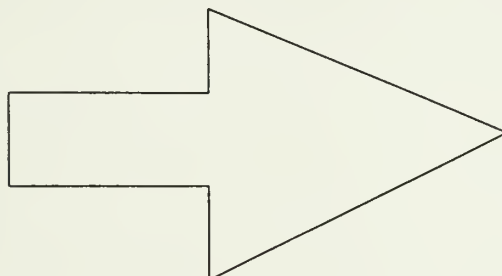
Question:

2. Given the following diagram of a specialized tool, and a ratio of 1:3, draw the arrowhead in its actual size and indicate its dimensions.



Solution:

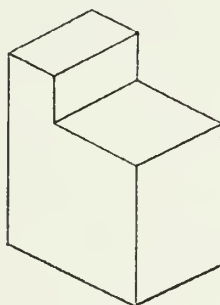
2.



Final dimensions will depend on the size of the initial drawing.

[✓]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

Question:



3. a. Given the 3-D object shown, draw an orthographic diagram of it.
b. If the ratio of the lengths of the object is 1:10, then make a cardboard or paper net and construct a 3-D representation of the actual object.
c. A 5 cm diameter hole needs to be bored from top to bottom, down the centre of the left taller section, and a 7.5 cm × 7.5 cm × 10 cm rectangular cutout needs to be made at the bottom right section of the actual object.
- Mark the location of the hole and the cut on your constructed object.
 - Draw an orthographic diagram of the object given above, with its hole and cutout, reduced in the ratio 1:0.8.

(continued)

General and Specific Outcomes
General Outcome Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. Specific Outcome 6.1 Enlarge or reduce a dimensioned object, according to a specified scale. [C, CN, PS, V]

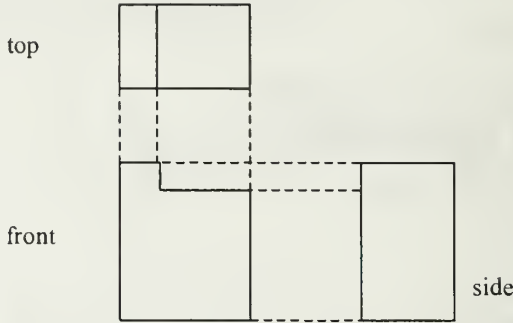
Sample Tasks

[✓]	Conceptual
[✓]	Procedural
[✓]	Problem-solving

(continued)

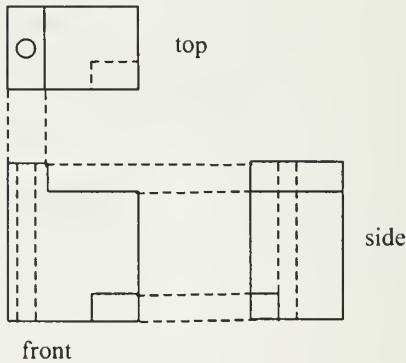
Solution:

3. a.



b. Students could redraw their orthographic drawings to actual size or take their dimensions and multiply by 10, to make their cardboard/paper cutouts.

c.



Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: provide solutions to questions 1 and 2, and question 3, parts a and b.	A student demonstrating the <i>standard of excellence</i> can also: complete question 3, part c.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

- 6.2 Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes.

[PS, R, V]

[C] Communication

[CN] Connections

[E] Estimation and

Mental Mathematics

[PS] Problem Solving

[R] Reasoning

[T] Technology

[V] Visualization

Notes:

- This specific outcome builds on outcomes in Applied Mathematics 10.
- Students have been introduced to the concept of the precision of a measuring instrument in Applied Mathematics 10.
- Plus and minus tolerances only apply to measured quantities; e.g., length, mass, capacity.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- calculate maximum and minimum values, using measured tolerances in routine/familiar problems
- calculate the sum or difference of two measurements, including measurement errors.

A student demonstrating the *standard of excellence* can also:

- calculate maximum and minimum values, using measured tolerances in nonroutine/unfamiliar problems
- calculate the sum or difference of more than two measurements, including measurement errors.

General and Specific Outcomes
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems. Specific Outcome 6.2 Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. [PS, R, V]

Sample Tasks

[✓]		Conceptual
[✓]		Procedural
[]		Problem-solving

Question:

- Measure the height of a styrofoam coffee cup, using a metric ruler with a precision of one millimetre.
 - State your answer, including the measurement error.
 - How many decimal places are in your answer? Why?

Solution:

- The height is approximately 8.30 ± 0.05 cm.
 - The answer has two decimal places, because the measurement error has two decimal places.

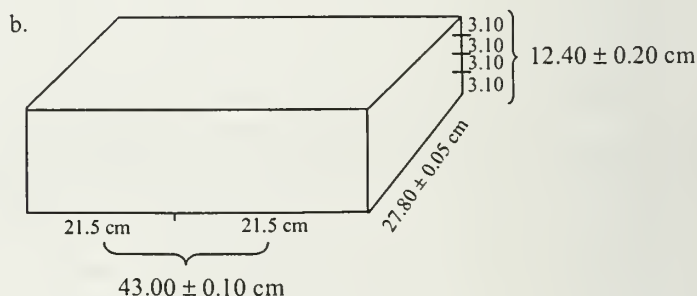
[✓] | Problem-solving

Question:

- A paper manufacturer wants to design a box to hold packages of printer paper. Each box is to hold 8 packages of paper stacked 4 high and 2 across. Each package of paper contains 200 sheets of paper and has the following dimensions: 27.8 cm $\times$ 21.5 cm $\times$ 3.1 cm. The packages will be placed side by side in the box so that the longer sides are adjacent.
 - State the length, width and height of each package of paper, with the measurement error.
 - Sketch a three-dimensional box with its measures that include measurement error.
 - Sketch an orthographic diagram of the box.
 - What should be the manufacturing dimensions of the box that will guarantee that the packages of paper will just fit?

Solution:

- The length is 27.80 ± 0.05 cm.
The width is 21.50 ± 0.05 cm.
The height is 3.10 ± 0.05 cm.



(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

- 6.2 Calculate maximum and minimum values, using tolerances, for lengths, areas and volumes. [PS, R, V]

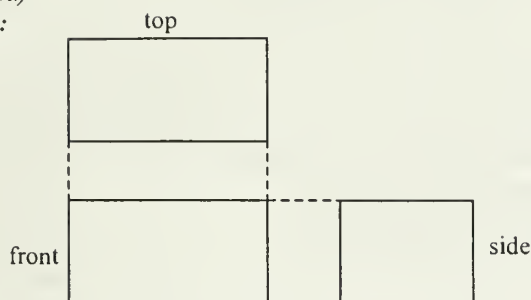
Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

(continued)

Solution:

2. c.



d. The dimensions of the box should be:

$$\text{Length: } 43.00 + 0.10 \text{ cm} = 43.10 \text{ cm}$$

$$\text{Width: } 27.80 + 0.05 \text{ cm} = 27.85 \text{ cm}$$

$$\text{Height: } 12.40 + 0.20 \text{ cm} = 12.60 \text{ cm.}$$

[✓] Procedural

Question:

3. A 10 L container, with a scale marking of 0.5 L, contains 2 L of orange juice concentrate. Determine the following:
- the volume of concentrate, including the measurement error
 - the measure of the volume of liquid in the container, if Zena adds 6.5 ± 0.05 L of water
 - the amount of orange juice that is left in the container, if Zena decides to sample the orange juice by pouring herself a 0.25 ± 0.005 L cup of juice.

Solution:

3. a. 2.00 ± 0.25 L of concentrate
- b. $(2.00 \pm 0.25)\text{L} + (6.5 \pm 0.05)\text{L}$
 $= 8.50 \pm 0.30$ L
 $= 8.5 \pm 0.3$ L of liquid
- c. $(8.5 \pm 0.3)\text{L} - (0.25 \pm 0.005)\text{L}$
 $= (8.25 \pm 0.305)\text{L}$
 $= 8.3 \pm 0.3$ L.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- provide an answer for both parts of question 1
- answer question 2, part a
- in question 2, part b, sketch the three-dimensional box with the sum of the measurement of each side indicated in some fashion
- answer all of question 3, except for the measurement errors.

A student demonstrating the *standard of excellence* can also:

- answer all parts of all questions correctly, including the related measurement errors.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

- 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

[C] Communication [PS] Problem Solving
 [CN] Connections [R] Reasoning
 [E] Estimation and [T] Technology
 Mental Mathematics [V] Visualization

Notes:

- This specific outcome builds on outcomes in Applied Mathematics 10.
- Students can relate their understanding of measurement error, from the previous specific outcome, to percentage error.
- The design problem found in the *Applied Mathematics 11 Source Book* (pp. 396–397) is solved incorrectly. The formula for percentage errors in this context should read $(\% \text{ error in } V) = (\% \text{ error in } d^2) + (\% \text{ error in } h)$.

Descriptions of Student Performance (Related to Specific Outcome)

A student demonstrating the *acceptable standard* can:

- convert between tolerances and percentage error
- solve simple problems involving tolerance and percentage error.

A student demonstrating the *standard of excellence* can also:

- solve complex/multistep problems involving tolerance and percentage error.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

Sample Tasks

		Conceptual
✓		Procedural
		Problem-solving

Question:

1. a. Find the percentage error for a length measurement of 10.50 ± 0.05 cm.
- b. Find the measurement error for a mass measurement of $23.5 \text{ g} \pm 2.5\%$, and express the mass with its measurement error.

Solution:

1. a. 10.50 ± 0.05 cm

Percentage of error is $\left(\frac{0.05}{10.5} \times 100\right)\% = 0.476\% \cong 0.48\%$

- b. The calculated measurement error is 2.5% of $23.5 \text{ g} = 0.5875 \text{ g}$.
The mass with its measurement error is $23.5 \pm 0.6 \text{ g}$.

[✓] | Conceptual

Question:

2. Compare the accuracy of the measurements $42.80 \pm 0.05 \text{ mL}$ and $6.530 \pm 0.005 \text{ L}$.

Solution:

2. Percentage error for

$$42.80 \pm 0.05 = \left(\frac{0.05}{42.80} \times 100\right)\% = 0.1168\% \cong 0.12\%$$

Percentage error for

$$6.530 \pm 0.005 = \left(\frac{0.005}{6.530} \times 100\right)\% = 0.07656\% \cong 0.077\%$$

Since the percentage error for the second measurement is less, it is more accurate.

(continued)

General and Specific Outcomes
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems. Specific Outcome 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [✓] | Procedural |
| [] | Problem-solving |

(continued)

Question:

3. A basketball court measures 94 feet by 48 feet. The length is measured with an error of 0.5%, and the width is measured with an error of 0.25%.
 - a. Find the percentage error in the calculated area of the basketball court.
 - b. Calculate the possible maximum and minimum areas of the basketball court.

Solution:

3. a. $\text{percentage error} = 0.5\% + 0.25\% = 0.75\%$
- b. Method 1:

$$(94 \times 48) \pm 0.75\% = 4512 \pm 0.75\% = 4512 \pm 34 \text{ ft}^2$$

Maximum area $4545.84 \approx 4546 \text{ ft}^2$
 Minimum area $4478.16 \approx 4478 \text{ ft}^2$

Method 2:

Maximum area $= (94 + 0.5\%) \times (48 + 0.25\%)$
 $= 94.47 \times 48.12$
 $= 4546 \text{ ft}^2$

Minimum area $= (94 - 0.5\%) \times (48 - 0.25\%)$
 $= 93.53 \times 47.88$
 $= 4478 \text{ ft}^2$

[✓] Problem-solving

Question:

4. Materials needed: pipe, metric ruler marked in millimetres, vernier calliper (metric).
 - a. Measure the length and inside diameter of the pipe, using the ruler. State the dimensions with the measurement error and with the percentage error.
 - b. Calculate the volume of the air contained within the pipe with the percentage error, using $V = \pi r^2 h$ ($V = \pi \cdot r \cdot r \cdot h$).
 - c. Measure the inside diameter of the pipe again, this time using the vernier calliper. State the dimensions with the percentage error.
 - d. Calculate the volume of the air contained within the pipe with the percentage error.
 - e. Explain the difference in the percentage error between the two volumes. What caused the change in the percentage error and why?

(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

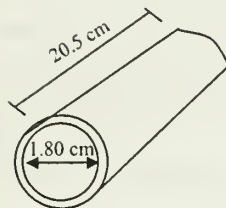
Specific Outcome

- 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

Sample Tasks

- | | |
|-----|-----------------|
| [] | Conceptual |
| [] | Procedural |
| [✓] | Problem-solving |

(continued)
Solution:



4. a. Length $20.50 \text{ cm} \pm 0.05 \text{ cm}$
 Inside diameter $1.80 \pm 0.05 \text{ cm}$
 Percentage error:
 length $\frac{0.05}{20.50} \times 100\% = 0.24\%$
 length $20.50 \text{ cm} \pm 0.24\%$
 diameter $\frac{0.05}{1.80} \times 100\% = 2.78\%$
 diameter $1.80 \text{ cm} \pm 2.78\%$
- b. radius $= 0.900 \pm 0.025 \text{ cm}$
 $\frac{0.025}{0.900} \times 100\% = 2.78\%$
 height $= 20.50 \pm 0.05 \text{ cm}$
 $\frac{0.05}{20.50} \times 100\% = 0.24\%$
 Volume percentage error $= 2.78\% + 2.78\% + 0.24\% = 5.80\%$
 Volume $= [\pi (0.9)^2 (20.5)] \pm 5.80\%$
 $= (52.17 \pm 5.80\%) \text{ cm}^3$
 $= 52.17 \pm 3.03 \text{ cm}^3$
 Volume $= 52 \pm 3 \text{ cm}^3$
- c. $d = (1.80 \pm 0.01) \text{ cm}$ with callipers
 $\ell = (20.50 \pm 0.05) \text{ cm}$ with ruler
 Percentage of error in $d = 0.56\%$
 Percentage of error in $\ell = 0.24\%$ as before
 $d = 1.80 \pm 0.56\%$
 $\ell = 20.50 \pm 0.24\%$
- d. Volume $= \pi (0.900 \text{ cm})^2 (20.50 \text{ cm})$
 $= 52.17 \text{ cm}^3$
 Percentage of error $= 0.56\% + 0.56\% + 0.24\%$
 $= 1.36\%$
 Volume $= (52.17 \pm 1.36\%) \text{ cm}^3$
 $= 52.2 \pm 0.7 \text{ cm}^3$
- e. Using the vernier calliper led to a smaller percentage error when calculating the volume of air in the pipe. The vernier calliper will measure the diameter of the pipe with greater precision, giving less measurement error and, therefore, less percentage error.

(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

- 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

Sample Tasks

[]	Conceptual
[]	Procedural
[✓]	Problem-solving

(continued)

Question:

5. A soup can is designed to have a volume of 287 ± 1 mL. The height is given as 10.12 ± 0.02 cm. The volume of the can of soup must never be less than 286 mL.
- Determine the diameter of the can, assuming a cylindrical shape.
 - Determine the tolerance limits for the diameter of the can.

Solution:

5. Solution 1 – using a percentage error formula

The volume of the can is given by the formula $V = \frac{1}{4}\pi d^2 h$.

Rearrange this formula in terms of d : $d = \sqrt{\frac{4V}{\pi h}}$.

Substitute middle values for V and h : $d = \sqrt{\frac{4(287 \text{ mL})}{\pi(10.12 \text{ cm})}}$ or
 $d = 6.009$ cm.

Now the percentage errors are linked as follows:

% error in $V = \text{% error in } d^2 + \text{% error in } h$

$$\frac{1}{287} \times 100\% = \text{% error in } d^2 + \frac{0.02}{10.12} \times 100\%$$

$$0.3484 = \text{% error in } d^2 + 0.1976$$

$$\text{% error in } d^2 = 0.3484 - 0.1976 = 0.1508$$

As % error in $d^2 = 2$ (% error in d), % error in $d = 0.0754$

So the design criteria for d has $d = 6.009 \pm 0.0754\%$, which is 6.009 ± 0.0045 cm.

This would be written as 6.009 ± 0.004 cm.

The 0.0045 cm is rounded down because the rounding process must reduce the allowed tolerance, not increase it.

Check to see if, with a minimum value of d and a minimum value of h , the volume is over 286 mL.

$$V = \frac{1}{4}\pi(6.005)^2(10.10) = 286.05 \text{ mL}$$

Thus the design will satisfy the requirement that the volume is never less than 286 mL.

(continued)

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

- 6.3 Solve problems involving percentage error when input variables are expressed with percentage errors using either formulas or first principles. [PS, R, V]

Sample Tasks

[]	Conceptual
[]	Procedural
[✓]	Problem-solving

(continued)

Solution:

5.

Solution 2 – using a first principles approach

The volume of the can is given by the formula $V = \frac{1}{4} \pi d^2 h$.

Rearrange this formula in terms of d : $d = \sqrt{\frac{4V}{\pi h}}$.

Substitute middle values for V and h : $d = \sqrt{\frac{4(287 \text{ mL})}{\pi(10.12 \text{ cm})}}$ or

$d = 6.009 \text{ cm}$.

This is the middle value for d .

To get the upper value of d , choose the highest value of V and the highest value of h .

$$d = \sqrt{\frac{4(288 \text{ mL})}{\pi(10.14 \text{ cm})}} = 6.0136$$

To get the lower value of d , choose the lowest value of V and the lowest value of h .

$$d = \sqrt{\frac{4(286 \text{ mL})}{\pi(10.10 \text{ cm})}} = 6.0045$$

To make sure that the design limits include less error than does the range from 6.0045 to 6.0136, the tolerances will only be plus or minus 0.004.

So the final design limits are $6.009 \pm 0.004 \text{ cm}$, or from 6.005 cm to 6.013 cm.

Descriptions of Student Performance (Related to Sample Tasks)

A student demonstrating the *acceptable standard* can:

- answer all of question 1 but makes rounding errors
- calculate one percentage error in question 2
- complete all of question 3
- find the measures of the dimensions with the measurement errors in question 4, using the ruler
- calculate the volumes in question 4
- calculate the value of d in question 5.

A student demonstrating the *standard of excellence* can also:

- answer all of question 1
- answer question 2
- find the inside diameter using the vernier calliper in question 4, part c
- state the volumes in question 4 with the correct percentage errors
- explain why in question 4 there is a change in the percentage error when using the vernier calliper to measure the diameter of the pipe
- calculate the tolerance limits for d in question 5.

General and Specific Outcomes
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems. Specific Outcome 6.4 Design an appropriate measuring process or device to solve a problem. [E, PS, V]

[C] Communication	[PS] Problem Solving
[CN] Connections	[R] Reasoning
[E] Estimation and Mental Mathematics	[T] Technology
	[V] Visualization

Notes: • This specific outcome lends itself to project work. • This specific outcome should be linked to the other specific outcomes in this topic. • The focus should be on different measuring strategies, rather than on the design of a device. • Design of measuring devices is more aligned with the standard of excellence.

Descriptions of Student Performance (Related to Specific Outcome)	
A student demonstrating the <i>acceptable standard</i> can: • select an appropriate process to solve a measurement problem • analyze the scale of a measuring instrument • design a measuring instrument, with guidance.	A student demonstrating the <i>standard of excellence</i> can also: • design an appropriate process to solve a measurement problem • design and analyze the scale of a measuring device • design a measuring instrument independently.

General and Specific Outcomes

General Outcome

Use measuring devices to make estimates and to perform calculations in solving problems.

Specific Outcome

6.4 Design an appropriate measuring process or device to solve a problem. [E, PS, V]

Sample Tasks

☐	☐	Conceptual
☐	☐	Procedural
☒	☐	Problem-solving

Question:

1. a. Given a ball bearing with a diameter of no more than 4 cm, describe two methods of measurement that would enable you to find the volume of the spherical object.
- b. Which method would give you a more accurate measurement? Justify your answer.

Solution:

1. a. Four possibilities are as follows:
 - Using callipers, find the diameter of the sphere. Then use $V = \frac{1}{6}\pi d^3$ to determine the volume.
 - Using a string, find the circumference of the sphere. Using $C = \pi d$ or $C = 2\pi r$, find either d or r . Then use $V = \frac{1}{6}\pi d^3$ or $V = \frac{4}{3}\pi r^3$ to determine the volume.
 - Use the water displacement method, using more than one identical bearing with a graduated cylinder or overflow can, assuming that the volume of water displaced is equal to the volume of the sphere.
 - If you have several of the same spheres. Use a grooved ruler to find the length of several spheres lined up side by side. From that, find the diameter of one sphere. Then use $V = \frac{1}{6}\pi d^3$ to determine the volume.
- b. Answers may vary, but justification should be related to tolerances and percentage error.

☒	☐	Conceptual
☒	☐	Procedural
☒	☐	Problem-solving

Question:

2. A technical designer wants to make a dip-stick, with scale markings, that measures the volume of gasoline in a cylindrical holding tank. To model this problem, the designer used a cylindrical frozen juice can, a graduated cylinder to measure volumes of water, and a popsicle stick.
 - a. Using the materials indicated, put a scale on a popsicle stick to measure the volume of water in the can when it is upright ().
 - b. Explain the pattern of the scale markings, and indicate the tolerance of your scale.
 - c. Using the materials indicated, put a scale on another popsicle stick to measure the volume of water in the can when it is on its side (). Make sure your lid is on securely. Put a small hole in the side of the can for inserting the popsicle stick.

(continued)

General and Specific Outcomes
General Outcome Use measuring devices to make estimates and to perform calculations in solving problems.
Specific Outcome 6.4 Design an appropriate measuring process or device to solve a problem. [E, PS, V]

Sample Tasks

[✓]		Conceptual
[✓]		Procedural
[✓]		Problem-solving

(continued)

Question:

2.
 - d. Compare the pattern of the scale markings on this stick with the one made in part a.
 - e. Would the tolerance of the scale on the stick made in part c be the same for all volume measures? Explain why or why not.
 - f. If the can in part c was half its original length, how would the scale on the popsicle stick change?

Solution:

2.
 - a. Students need to use volumes of water in regular increments; e.g., 50 mL, when marking their scale. They can also test the scale by measuring volumes of water and comparing these measurements to similar measurements made using graduated cylinders.
 - b. The scale pattern should be linear since the volume increases directly with the height. A method for determining the tolerance would be to measure volumes of unknown quantities of water and compare the measured values to their measured amounts in a graduated cylinder.
 - c. Scale markings should be spaced further apart near the top and the bottom of the scale.
 - d. The scale markings on the popsicle stick in part c will not be linear, since the volume is affected by the curvature of the cylinder.
 - e. No. The tolerance close to the centre of the can should be higher since the level changes less than when the can is almost empty or full.
 - f. The scale markings would be the same, but the volume measured at each position would be half the amount of the original can.

Descriptions of Student Performance (Related to Sample Tasks)	
A student demonstrating the <i>acceptable standard</i> can: - answer question 1, parts a and b, with very weak justification - provide answers to question 2, parts a, b and c, with guidance.	A student demonstrating the <i>standard of excellence</i> can also: - answer question 1, part b, with an appropriate justification; and in question 1, part a, the student might develop a unique method of measurement - provide answers to question 2, parts d, e and f.

